Name: _____ CompID: _____

Theorem 1 $\left(1 + \sum_{i=0}^{n-1} 2^i\right) = 2^n$

Theorem 2 *A graph* $G = (V, E)$*, where* $V = \{v \mid (1 \leq v \leq n) \wedge (v \in \mathbb{N})\}$ *and* $E = \{(v, v+1) \mid (v \in V) \wedge (v+1 \in V)\}$ *has* $\left(\sum_{i=1}^{n-1} i\right)$ *distinct paths*

PROBLEM 1 *Prove* ***one*** *of theorems 1–2 (your choice which one) using* ***induction***

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Theorem 1 $\log_5(7) \notin \mathbb{Q}$ **Theorem 2** $\sqrt[3]{11} \notin \mathbb{Q}$ **Theorem 3** $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

PROBLEM 1 *Prove **one** of theorems 1–3 (your choice which one) using* ***contradiction***

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PROBLEM 1 *Fill in these* ***combinatorics*** *blanks*

You may answer any question with factorial, choose, and unresolved arithmetic notation, but may not use ellipses. For example, the following are all OK: $\boxed{120}$, $\boxed{5!}$, $\boxed{\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1) + (3 \cdot 2 \cdot 1)}}$, $\boxed{\binom{5}{3}}$.

1. _____ An economy license plate number starts with X or W, then two more letters (out of 24 options, not 26, because O and I are not used), then four digits (all ten used). Repetition is allowed (e.g., "WWX 0000" is OK). How many license plate numbers can this scheme create?

2. _____ How many 9-element subsets of a 100-element set are there?

3. _____ I draw 3 cards from a deck of 50 distinct cards and line them up in a row. How many distinct rows of cards could I get?

4. _____ Which is larger: $\binom{45}{40}$ or $\binom{45}{42}$?

5. _____ A palindrome is a string is the same if you reverse it, like "rrynyrr". How many 7-letter palindromes can be made from the set of 26 letters?

6. _____ I randomly shuffle a list containing five `"d"`s, five `"q"`s, and five `"w"`s,. What is the probability the shuffle will result in the exact sequence `"dqwdqwdqwdqwdqw"`?

7. _____ I randomly shuffle a 15-item list containing the integers 1 through 15. What is the probability the shuffle will result in the exact sequence $(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$?

8. _____ I roll a pair of twenty-sided dice, with sides numbered 1 through 20. What is the probability at least one die will roll a 6?

9. _____ You can either choose a 5-letter string (out of 26 letters, which can repeat; e.g `"xyxxy"` or `dfghj` are both options) or a 5-digit number (out of ten digits, which can repeat, but the first digit must not be 0; e.g. `21020` is an option but `02102` is not). How many options do you have?

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Consider the following **sets**: $A = \{2, 4, 8\}$, $B = \{1, 2, 4\}$, $C = \mathcal{P}\big(\{1, 2\}\big)$

PROBLEM 1 *Show all members of each set*

1. _____ $= C$
2. _____ $= A \cup B$
3. _____ $= A \cap B$
4. _____ $= A \setminus B$
5. _____ $= \big\{x \;\big|\; (x \in \mathbb{N}) \wedge (2x \in B)\big\}$
6. _____ $= B \cap C$
7. _____ $= \big\{x \;\big|\; (x \in A) \wedge (2x \in B)\big\}$
8. _____ $= \Big\{\{a, b\} \;\Big|\; (a \in A) \wedge \big(b \in \{4, 8\}\big)\Big\}$

PROBLEM 2 *Answer each question*

9. _____ $= \big|\{1, 2, 3, 4\}\big|$
10. _____ $= \big|\mathcal{P}(A)\big|$
11. _____ $= \Big|\mathcal{P}\Big(\mathcal{P}\big(\{1, 2\}\big)\Big)\Big|$
12. _____ $= 8 \in C$
13. _____ $= \{8\} \in C$
14. _____ $= \big\{\{8\}\big\} \in C$

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Consider the following **discrete structures** questions.

PROBLEM 1 *Write out in full*

1. _____ $= \{1,2\} \times \{1\} \times \{1,2\}$

2. _____ $= \left|\{1,2,3\}^4\right|$

3. _____ = all the subsequences of `"OK"`

4. _____ = a subsequence of `"xyxxy"` that is not a substring of `"xyxxy"`

5. _____ = the image of $\{0,3\}$ under $R(x) = x + 2$

6. _____ = the set of edges of the graph ①→②⇄③

PROBLEM 2 *Draw*

7. ○⇄○←○ add a minimal number of edges to make this the graph of a transitive relation

8. ○⇄○←○ add a minimal number of edges to make this the graph of a symmetric relation

PROBLEM 3 *Logarithms*

9. Simplify $\log_5(24) - \log_5(4)$: _____

10. Re-write $\log_{10}(x^7)$ without exponentiation: _____

11. Re-write $\log_w(8)$ using base-x log(s) instead of base-w: _____

12. Fill in the blank: $\log_9(4) = \log_3\left(\_\_\_\_\_\right)$

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Consider the following **logic** questions. You do not need to specify your domains, propositions, or predicate definitions, though you may if you wish.

PROBLEM 1 *Convert the underlined parts to logic*

1. "can any sorting algorithm be faster than quicksort?"

2. "Every klaxon is a horn."

3. "No patents apply to this code"

4. "Some 100-digit number is prime."

PROBLEM 2 *Convert to English*

domain: all animals
$M(x)$: $\_\_\_\__x$ is a monkey
$L(x, y)$: $\_\_\_\__x$ loves $\_\_\_\__y$

5. Write a clear English sentence that means $\exists x \,.\, \forall y \neq x \,.\, L(y, x)$.

6. Write a clear English sentence that means $\forall x \,.\, M(x) \to \big(\exists y \,.\, L(x, y) \wedge L(y, x)\big)$.

(continued on reverse)

PROBLEM 3 *Apply axioms*

Show that $\big((P \wedge Q) \rightarrow W\big) \equiv \big(P \rightarrow (W \vee \neg Q)\big)$ by direct proof and/or proof by cases. You may mix math and English if you wish; we are looking for sound logic, not prose proof technique.