Name: ______________________________ CompID: ______________

CS 2102 - DMT1 - Fall 2019 — Luther Tychonievich
Administered in class Friday November 8, 2019

Quiz 09

Theorem 1 $\forall x \in \{a \mid a \in \mathbb{Z} \wedge a \geq -1\} \,.\, \sum_{k=-1}^{x} 12 - 2k = 26 + 11x - x^2$

PROBLEM 1 *Proof by Induction*

Prove the above theorem using induction.

Proof.

We proceed by induction.

Base Case When $x = -1$ we have $\sum_{k=-1}^{-1} 12 - 2k = 14 = 26 - 11 - 1$, so the theorem holds for $x = -1$.

Inductive step Assume the theorem holds for some x; that is, $\sum_{k=-1}^{x} 12 - 2k = 26 + 11x - x^2$. Consider the sum evaluated at $x + 1$:

$$\begin{aligned}
\sum_{k=-1}^{x+1} 12 - 2k &= 12 - 2(x+1) + \sum_{k=-1}^{x} 12 - 2k \\
&= 11 - 2x + 26 + 11x - x^2 \\
&= 26 + (11 + 11x) - (1 + 2x + x^2) \\
&= 26 + 11(x+1) - (x+1)^2
\end{aligned}$$

which means the theorem holds at $x + 1$ as well.

By the principle of induction, the theorem holds for all $x \in \{a \mid a \in \mathbb{Z} \wedge a \geq -1\}$.

□