Name: ____ CompID: ____________________

CS 2102 - DMT1 - Fall 2019 — Luther Tychonievich
Administered in class friday november 22, 2019

Quiz 11

Theorem 1 *The shortest walk between any pair of vertices is a path.*

Prove Theorem 1, using proof by contradiction.
Proof.

We proceed by contradiction.

Assume there exists some pair of vertices, a and b, where the shortest walk w between them is not a path. Then w must visit some vertex, v, more than once.

Let i be the index of the first occurrence of v in w and j be the index of the last occurrence of v in w. Because v appears more than once, $i < j$.

Let w' be a walk defined as the first i elements of w followed by the elements of w after j. By construction, w' starts at a and ends and b. Because $i < j$, $|w'| < |w|$. But this contradicts the assertion that w is the shortest walk between a and b.

Because assuming the existence of a non-path shortest walk led to a contradiction, there must not be any non-path shortest walks. Hence, every shortest walk is a path.

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