Practice 02

problem 1 *Equivalence rules*

Consider the following equivalence rules:

- the associativity and commutativity of $\wedge$, $\vee$, and $\oplus$
- double negation: $\boxed{\neg\neg P \equiv P}$
- simplification: $\boxed{P \wedge \bot \equiv \bot}$, $\boxed{P \wedge \top \equiv P}$, $\boxed{P \vee \bot \equiv P}$, and $\boxed{P \vee \top \equiv \top}$
- distribution: $\boxed{A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)}$ and $\boxed{A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)}$
- De Morgan: $\boxed{\neg(A \wedge B) \equiv (\neg A) \vee (\neg B)}$ and $\boxed{\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)}$
- definitions: $\boxed{A \rightarrow B \equiv (\neg A) \vee B}$, $\boxed{(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A)}$ and $\boxed{(A \oplus B) \equiv (A \vee B) \wedge \neg(A \wedge B)}$

Prove that $\boxed{P \rightarrow (A \vee Q) \equiv (P \wedge \neg A) \rightarrow Q}$ by writing out a series of steps, one per line, where the first line is $\boxed{P \rightarrow (A \vee Q)}$, the last line is $\boxed{(P \wedge \neg A) \rightarrow Q}$, and each line other than the first is an application of **one** of those equivalences to the line above it.

You do not need to name your steps, though doing so might help us grade more easily

Want more practice? Try doing the same for $\boxed{A \oplus B \equiv \overline{A \leftrightarrow B}}$ *and* $\boxed{A \leftrightarrow B \equiv \overline{A \oplus B}}$ *and, if you feel ambitious,* $\boxed{A \oplus B \oplus C \equiv (A \wedge \overline{B} \wedge \overline{C}) \vee (\overline{A} \wedge B \wedge \overline{C}) \vee (\overline{A} \wedge \overline{B} \wedge C) \vee (A \wedge B \wedge C)}$

PROBLEM 2 *Prose proof by case analysis*

Write a prose proof of $\boxed{P \oplus Q \equiv \overline{P} \oplus \overline{Q}}$ by completing the provided proof-by-cases template.

Proof. Either ________ is true or it is false.

Case 1: ________ is true The expression $P \oplus Q$ in this case

The expression $\overline{P} \oplus \overline{Q}$ in this case

Because the two are equivalent to the same thing, they are equivalent to each other.

Case 2: ________ is false The expression $P \oplus Q$ in this case

The expression $\overline{P} \oplus \overline{Q}$ in this case

Because the two are equivalent to the same thing, they are equivalent to each other.

Because $P \oplus Q \equiv \overline{P} \oplus \overline{Q}$ is true in both cases, it is true in general. □