Name: ____ CompID: ____________________

CS 2102 - DMT1 - Fall 2019 — Luther Tychonievich
Administered in class friday september 13, 2019

Quiz 02

problem 1 *Equivalence rules*

Consider the following equivalence rules:

- the associativity and commutativity of $\wedge$, $\vee$, and $\oplus$
- double negation: $\boxed{\neg\neg P \equiv P}$
- simplification: $\boxed{P \wedge \bot \equiv \bot}$, $\boxed{P \wedge \top \equiv P}$, $\boxed{P \vee \bot \equiv P}$, and $\boxed{P \vee \top \equiv \top}$
- distribution: $\boxed{A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)}$ and $\boxed{A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)}$
- De Morgan: $\boxed{\neg(A \wedge B) \equiv (\neg A) \vee (\neg B)}$ and $\boxed{\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)}$
- definitions: $\boxed{A \rightarrow B \equiv (\neg A) \vee B}$, $\boxed{(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A)}$ and $\boxed{(A \oplus B) \equiv (A \vee B) \wedge \neg(A \wedge B)}$

Prove that $\boxed{(P \wedge \neg Q) \equiv \neg(P \rightarrow Q)}$ by writing out a series of steps, one per line, where the first line is $\boxed{(P \wedge \neg Q)}$, the last line is $\boxed{\neg(P \rightarrow Q)}$, and each line other than the first is an application of **one** of those equivalences to the line above it.

You do not need to name your steps, though doing so might help us grade more easily

$(P \wedge \neg Q)$
$(\neg\neg P \wedge \neg Q)$
$\neg(\neg P \vee Q)$
$\neg(P \rightarrow Q)$

PROBLEM 2 *Prose proof by case analysis*

Write a prose proof of $(P \wedge Q) \rightarrow M \equiv P \rightarrow (Q \rightarrow M)$ by completing the provided template.

Proof. Either P is true or it is false.

Case 1: P **is true** The expression $(P \wedge Q) \rightarrow M$ in this case

can be simplified to $Q \rightarrow M$ by the equivalence of $\top \wedge Q$ and Q.

The expression $P \rightarrow (Q \rightarrow M)$ in this case

can be simplified to $Q \rightarrow M$ by the equivalence of $\top \rightarrow Q$ and Q.

Because the two are equivalent to the same thing, they are equivalent to each other.

Case 2: P **is false** The expression $(P \wedge Q) \rightarrow M$ in this case

can be simplified to $\bot \rightarrow M$ by the equivalence of $\bot \wedge Q$ and $\bot$, which in turn is just $\top$ regardless of the value of M.

The expression $P \rightarrow (Q \rightarrow M)$ in this case

is $\bot \rightarrow (Q \rightarrow M)$, which is $\top$ regardless of the values of Q and M.

Because the two are equivalent to the same thing, they are equivalent to each other.

Since $(P \wedge Q) \rightarrow M \equiv P \rightarrow (Q \rightarrow M)$ is true in both cases, it is true in general. □