CS 2102 - DMT1 - SPRING 2020 — LUTHER TYCHONIEVICH
PRACTICE EXERCISE IN CLASS FRIDAY FEBRUARY 14, 2020

PRACTICE 04

PROBLEM 1 *Symbolizing*

Provide a logic translation for each of the following.

1. Every sorting algorithm that is asymptotically faster than mergesort is limited in what kinds of elements can be in its list.
domain: algorithms

$S(x)$: x is a sorting algorithm
$M(x)$: x is faster than mergesort
$U(x)$: works on lists of any kind

$$\forall x \,.\, (S(x) \wedge M(x)) \rightarrow \neg U(x)$$

2. I love everyone who loves me as long as they also like peanut butter or cheddar cheese on their lemon sorbet.
domain: people

$L(x, y)$: x loves y
$P(x)$: x likes peanut butter on lemon sorbet
$C(x)$: x likes cheddar cheese on lemon sorbet
m: Me

$$\forall x \,.\, \big(L(x, m) \wedge (P(x) \vee C(x))\big) \rightarrow L(m, x)$$

What more practice? See Practice Quiz 03 for a list from our textbooks

PROBLEM 2 *Prosify*

Convert the following proof outlines into prose proofs.

3. Theorem: $(P \wedge Q) \rightarrow R \equiv P \rightarrow (R \vee \neg Q)$
Proof outline:
$(P \wedge Q) \rightarrow R \equiv \neg(P \wedge Q) \vee R \equiv (\neg P \vee \neg Q) \vee R \equiv \neg P \vee (\neg Q \vee R) \equiv \neg P \vee (R \vee \neg Q) \equiv P \rightarrow (R \vee \neg Q)$

Proof.

$(P \wedge Q) \rightarrow R$ is equivalent to $\neg(P \wedge Q) \vee R$ by the definition of implication; De Morgan's law changes that to $(\neg P \vee \neg Q) \vee R$, which is the same as $\neg P \vee (R \vee \neg Q)$ by the associative and commutative properties of disjunction. Using the definition of implication again, we arrive at $P \rightarrow (R \vee \neg Q)$.

□

4. Theorem: There is an integer that every other integer divides.
Formalism: $\exists x \,.\, \forall y \,.\, D(x,y)$ where $D(x,y)$ means "x divides y".
Proof outline: $x = 1$; $y \div 1 = y$ remainder 0; $\therefore \exists x \,.\, \forall y \,.\, D(x,y)$

Proof.

Consider $x = 1$. 1 divides y with no remainder regardless of what y is. Thus, $\forall y \,.\, D(1,y)$. Since this works for $x = 1$, we know that $\exists x \,.\, \forall y \,.\, D(x,y)$.

□

What more practice? Take any proof from Quiz 02 or our textbook and try converting to prose

PROBLEM 3 *Complete*

Fill in the blanks to complete the following proofs by cases.

Theorem 1 $(P \wedge Q) \rightarrow M \equiv P \rightarrow (Q \rightarrow M)$

Proof. 5. Either P is true or it is false.

Case 1: P **is true** The expression $(P \wedge Q) \rightarrow M$ in this case

6. can be simplified to $Q \rightarrow M$ by the equivalence of $\top \wedge Q$ and Q.

The expression $P \rightarrow (Q \rightarrow M)$ in this case

7. can be simplified to $Q \rightarrow M$ by the equivalence of $\top \rightarrow Q$ and Q.

Because the two are equivalent to the same thing, they are equivalent to each other.

Case 2: P **is false** The expression $(P \wedge Q) \rightarrow M$ in this case

8. can be simplified to $\bot \rightarrow M$ by the equivalence of $\bot \wedge Q$ and $\bot$, which in turn is just $\top$ regardless of the value of M.

The expression $P \rightarrow (Q \rightarrow M)$ in this case

9. is $\bot \rightarrow (Q \rightarrow M)$, which is $\top$ regardless of the values of Q and M.

Because the two are equivalent to the same thing, they are equivalent to each other.

Since $(P \wedge Q) \rightarrow M \equiv P \rightarrow (Q \rightarrow M)$ is true in both cases, it is true in general. □

Theorem 2 $P \oplus Q \equiv \overline{P} \oplus \overline{Q}$

Proof. 10. Either P is true or it is false.

Case 1: P **is true** The expression $P \oplus Q$ in this case

11.

is $\top \oplus Q$, which is defined to mean $(\top \vee Q) \wedge \overline{(\top \wedge Q)}$.
Simplifying, that is equivalent to $\top \wedge \overline{Q}$, or simply $\overline{Q}$.

The expression $\overline{P} \oplus \overline{Q}$ in this case

12.

is $\bot \oplus \overline{Q}$, which is defined to mean $(\bot \vee \overline{Q}) \wedge \overline{(\bot \wedge \overline{Q})}$.
We can simplify that to $(\overline{Q}) \wedge \overline{(\bot)}$,
which is equivalent to $\overline{Q} \wedge \top$ or simply $\overline{Q}$.

Because the two are equivalent to the same thing, they are equivalent to each other.

Case 2: P **is false** The expression $P \oplus Q$ in this case

13.

is $\bot \oplus Q$, which is defined to mean $(\bot \vee Q) \wedge \overline{(\bot \wedge Q)}$.
Simplifying, that is equivalent to $Q \wedge \top$, or simply Q.

The expression $\overline{P} \oplus \overline{Q}$ in this case

14.

is $\top \oplus \overline{Q}$, which is defined to mean $(\top \vee \overline{Q}) \wedge \overline{(\top \wedge \overline{Q})}$.
Simplifying, that is equivalent to $\top \wedge \overline{\overline{Q}}$, which is equivalent to Q.

Because the two are equivalent to the same thing, they are equivalent to each other.

Because $P \oplus Q \equiv \overline{P} \oplus \overline{Q}$ is true in both cases, it is true in general. □

What more practice? Try MCS problem 1.7; writing the example proofs in ∀x 15.6, 16.3, 19.2, and 19.6 in prose; ∀x practice 15.A and 15.B