Name: ____ CompID: ____________________

CS 2102 - DMT1 - Fall 2020 — Luther Tychonievich
Administered in class friday april 10, 2020

Quiz 10

problem 1 *Logic*

Use the following definitions for items 1–4 below:

- P: The set of programs
- T: The set of testing plans
- $B(p)$: Program p has a bug
- $B(p, t)$: Testing plan t reports program p has a bug

1. A *conservative* test plan never reports bugs unless bugs actually exist. A *complete* test plan always reports a bug if one exists. Write logic that means "some test plans are neither conservative nor complete."

$$\exists t \in T \;.\; \big(\exists p \in P \;.\; B(p,t) \wedge \neg B(p)\big) \wedge \big(\exists p \in P \;.\; B(p) \wedge \neg B(p,t)\big)$$

2. A *perfect* test plan reports bugs when they exist and only when they exist. A *universal* test plan works on all programs. Write logic that means "there's no perfect universal test plan."

$$\nexists t \in T \;.\; \forall p \;.\; B(p) \leftrightarrow B(p,t)$$

3. Convert this logic to English, clearly enough we can tell if you got the quantifiers in the right order:

$$\forall t \in T \;.\; \exists p \in P \;.\; B(p) \leftrightarrow B(p,t)$$

Every test plan has some program it handles perfectly.

4. Convert this logic to English, clearly enough we can tell if you got the quantifiers in the right order:

$$\exists t \in T \;.\; \forall p \in P \;.\; B(p) \leftrightarrow B(p,t)$$

There is a perfect universal test plan.

PROBLEM 2 *Functions*

5. Give an example function $f : \mathbb{Q} \to \mathbb{Q}$ which is **total** and **injective** (one-to-one) but not **surjective** (not onto). You are welcome to describe it using pseudo-code, math, or any other unambiguous format we will understand.

Given a rational number that has reduced form $\pm\frac{x}{y}$, return $\pm\frac{2^x}{3^y}$ (with the same sign as the input). By the uniqueness of prime factorization, this is one-to-one; but it does not cover most fractions (e.g., $\frac{1}{2}$).

6. Give an example function $f : \mathbb{Q} \to \mathbb{N}$ which is **total** and **surjective** (onto). You are welcome to describe it using pseudo-code, math, or any other unambiguous format we will understand.

Given a rational number that has reduced form $\pm\frac{x}{y}$, return $|x|$.