CS 2102 - DMT1 - Spring 2020 — Luther Tychonievich
Practice exercise in class friday february 14, 2020

# Practice 04

## problem 1 *Symbolizing*

Provide a logic translation for each of the following.

1. Every sorting algorithm that is asymptotically faster than mergesort is limited in what kinds of elements can be in its list.
domain: algorithms

2. I love everyone who loves me as long as they also like peanut butter or cheddar cheese on their lemon sorbet.
domain: people

*What more practice? See Practice Quiz 03 for a list from our textbooks*

## problem 2 *Prosify*

Convert the following proof outlines into prose proofs.

3. Theorem: $(P \wedge Q) \rightarrow R \equiv P \rightarrow (R \vee \neg Q)$
Proof outline:
$(P \wedge Q) \rightarrow R \equiv \neg(P \wedge Q) \vee R \equiv (\neg P \vee \neg Q) \vee R \equiv \neg P \vee (\neg Q \vee R) \equiv \neg P \vee (R \vee \neg Q) \equiv P \rightarrow (R \vee \neg Q)$

*Proof.*
□

4. Theorem: There is an integer that every other integer divides.
Formalism: $\exists x \,.\, \forall y \,.\, D(x, y)$ where $D(x, y)$ means "$x$ divides $y$".
Proof outline: $x = 1$; $y \div 1 = y$ remainder 0; $\therefore \exists x \,.\, \forall y \,.\, D(x, y)$

*Proof.*
□

*What more practice? Take any proof from Quiz 02 or our textbook and try converting to prose*

PROBLEM 3 *Complete*

Fill in the blanks to complete the following proofs by cases.

**Theorem 1** $(P \wedge Q) \rightarrow M \equiv P \rightarrow (Q \rightarrow M)$

*Proof.* 5. Either ________ is true or it is false.

**Case 1: ________ is true** The expression $(P \wedge Q) \rightarrow M$ in this case

6.

The expression $P \rightarrow (Q \rightarrow M)$ in this case

7.

Because the two are equivalent to the same thing, they are equivalent to each other.

**Case 2: ________ is false** The expression $(P \wedge Q) \rightarrow M$ in this case

8.

The expression $P \rightarrow (Q \rightarrow M)$ in this case

9.

Because the two are equivalent to the same thing, they are equivalent to each other.

Since $(P \wedge Q) \rightarrow M \equiv P \rightarrow (Q \rightarrow M)$ is true in both cases, it is true in general. □

**Theorem 2** $P \oplus Q \equiv \overline{P} \oplus \overline{Q}$

*Proof.* 10. Either ________ is true or it is false.

**Case 1: ________ is true** The expression $P \oplus Q$ in this case

11.

The expression $\overline{P} \oplus \overline{Q}$ in this case

12.

Because the two are equivalent to the same thing, they are equivalent to each other.

**Case 2: ________ is false** The expression $P \oplus Q$ in this case

13.

The expression $\overline{P} \oplus \overline{Q}$ in this case

14.

Because the two are equivalent to the same thing, they are equivalent to each other.

Because $P \oplus Q \equiv \overline{P} \oplus \overline{Q}$ is true in both cases, it is true in general. □

*What more practice? Try MCS problem 1.7; writing the example proofs in ∀x 15.6, 16.3, 19.2, and 19.6 in prose; ∀x practice 15.A, 15.B, and 15.C*