CS 2102 - DMT1 - Spring 2020 — Luther Tychonievich
Practice exercise in class friday february 28, 2020

Practice 06

problem 1 *Convert to prose*

Convert the following symbolic proof that `f(x)` = x^2 to prose.

```
let f(x) be computed as
    if x <= 0 then return 0
    else return (2*x-1) + f(x-1)
```

Symbolic Proof.

Line	Subproof line	Statement	Justification
1		$f(0) = 0 = 0^2$	definition
2	2	$f(x-1) = (x-1)^2$	assumption
	3	$f(x) = 2x - 1 + f(x-1)$	definition
	4	$f(x) = 2x - 1 + (x-1)^2$	combine line 2 and 3
	5	$f(x) = 2x - 1 + (x^2 - 2x + 1)$	algebra on line 4
	6	$f(x) = x^2$	simplify line 5
3		$\forall x \geq 0 \,.\, f(x) = x^2$	principle of induction on lines 1 and 2

Proof.

PROBLEM 2 *Code termination*

Prove by induction that each of the following functions terminate given any integer argument.

2.
```
let f(x) be computed as
    if x <= 0 then return x
    otherwise return 1 + f(x-1)
```
Proof.

3.
```
let f(x) be computed as
    if x <= 1 then return x
    otherwise return 1 + f(x-1) + f(x-2)
```
Proof.

4.
```
let f(x) be computed as
    if x >= -1 then return x
    otherwise return 1 + f(x+1)
```
Proof.

PROBLEM 3 *Code property*

Prove by induction each of the following functions returns an even number given any non-negative integer argument.

5.
```
let f(x) be computed as
    y = 0
    repeat x times:
        y += 2
    return y
```
Proof.

6.
```
let f(x) be computed as
     if x <= 0 then return 0
     else return 4 * f(x-1)
```
Proof.

7.
```
let f(x) be computed as
     if x <= 0 then return 2
     else return 2 * f(x-2)
```
Proof.