CS 2102 - DMT1 - Spring 2020 — Luther Tychonievich
Practice exercise in class friday march 6, 2020

Practice 07

problem 1 *Products and Powers*

Write out the following in full.

1. $\{1,2\} \times \{3\} \times \{1,4\} =$ ______________________________

2. $\{56\}^3 =$ ______________________________

3. $\{1,2\} \times \mathcal{P}(\{1\}) =$ ______________________________

4. $\{(1,2)\} \times \mathcal{P}(\{(1,2,3,4)\}) =$ ______________________________

5. $\{\mathsf{a}, \mathsf{b}\}^2 =$ ______________________________

6. $\{4,1\} \times \{1,2\} =$ ______________________________

7. $\{4\} \times \{1,2\} \times \{3\}^3 =$ ______________________________

8. $\mathcal{P}(\{\})^2 =$ ______________________________

problem 2 *Members of Products and Powers*

Give two different example members of each of the following sets. Make them different from one another: different lengths, different internal patterns, etc., is the set allows that. If there are not enough elements of the set to give two different elements, leave some blanks blank.

9. $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^4$ contains ______________ and ______________

10. $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^1$ contains ______________ and ______________

11. $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^0$ contains ______________ and ______________

12. $\{\mathsf{a}, \mathsf{b}, \mathsf{c}\}^*$ contains ______________ and ______________

13. {"`good`", "`fun`"}^{2} contains ____________________ and ____________________

Give two strings of length 3 belonging to

14. {"`a`", "`ok`"}*: _____________ and _____________

15. {"`a`", "`bb`", "`ccc`"}*: _____________ and _____________

PROBLEM 3 *Subsequences*

Definition 1 *A* ***subsequence*** *is a sequence that can be derived from another sequence by deleting zero or more elements without changing the order of the remaining elements.*

What are the subsequences of the string "`OK`"? ____

What is the longest subsequence shared by "`MATHEMATICS`" and "`COMPUTERS`"? ____________________

PROBLEM 4 *Summation proofs*

Prove the following theorems by induction.

16. $\forall n \in \mathbb{N} \; . \; \sum_{i=0}^{n} i = \frac{(n)(n+1)}{2}$

Proof.

17. $\forall n \in \mathbb{N} \ . \ \sum_{x=0}^{n} \frac{1}{2^x} = \frac{2^{n+1} - 1}{2^n}$

Proof.

18. $\forall n \in \mathbb{N} \,.\, \sum_{x=n}^{2n} x = \frac{3(n+1)n}{2}$

Proof.

19. $\forall x \in \{a \mid a \in \mathbb{Z} \land a \geq -1\} \,.\, \sum_{k=-1}^{x} 12 - 2k = 26 + 11x - x^2$

Proof.

You might also try doing inductive proofs with other summation formulae, such as

$$\begin{aligned}
\sum_{i=0}^{n} i^2 &= \frac{(n+1)(2n+1)(n)}{6} \\
\sum_{i=1}^{n+1} i^2 &= \frac{(n+2)(2n+3)(n+1)}{6} \\
\sum_{i=2}^{n+2} i^2 &= \frac{(n+3)(2n+5)(n+2)}{6} \\
6\sum_{i=0}^{n} i^3 - i &= \binom{n+2}{4} \\
\sum_{x=0}^{n} \frac{x^2-1}{x+1} &= \frac{(n+1)(n-1)}{2} \\
\sum_{x=0}^{n} x^3 - x^2 &= \frac{(n+1)(3n+2)(n)(n-1)}{12} \\
\sum_{i=0}^{n} 3i^2 + 2i &= \frac{(2n+3)(n+1)(n)}{2} \\
\sum_{x=n}^{n^2} x &= \frac{n+n^4}{2} \\
\sum_{x=0}^{2n} (-1)^x x &= n \\
\sum_{i=1}^{n} \frac{1}{2^i} &= \frac{2^n-1}{2^n} \\
\sum_{k=-n}^{0} k &= \frac{(n+1)n}{-2} \\
\sum_{i=1}^{n} \frac{1}{3^i} &= \frac{3^n-1}{3^n 2} \\
\forall k \neq 1 \,.\, \left(\sum_{i=1}^{n} \frac{1}{k^i} \right. &= \left. \frac{k^n-1}{k^n(k-1)}\right)
\end{aligned}$$

Note: at least one of the above formulae is false. In the process of proving it you should find the normal methods not working, revealing the non-truth.