Practice 10

problem 1 *Convert to prose*

P: the set of all single-input functions
I: the set of all inputs
$C(p, i)$: p crashes when run on i

Convert the following to simple, readable English. Make sure your answer shows how the questions are different:

1. $\exists p \in P \,.\, \forall i \in I \,.\, C(p, i)$

There's a program that crashes no matter what input you give it.

2. $\exists i \in I \,.\, \forall p \in P \,.\, C(p, i)$

There's one special input that will crash any program you run it on.

3. $\forall p \in P \,.\, \exists i \in I \,.\, C(p, i)$

Every program has some input it crashes on.

4. $\forall i \in I \,.\, \exists p \in P \,.\, C(p, i)$

Every input has some program it crashes.

Convert the following to logic:

5. If a program crashes on any input, it crashes on more than one input.

$\forall p \in P \,.\, \big(\exists i \in I \,.\, C(p, i)\big) \rightarrow \big(\exists i, j \in I \,.\, i \neq j \wedge C(p, i) \wedge C(p, j)\big)$

6. No program crashes on every input.

$\forall p \in P \,.\, \exists i \in I \,.\, \neg C(p, i)$

— *or* —

$\nexists p \in P \,.\, \forall i \in I \,.\, C(p, i)$

PROBLEM 2 *Identify domain and range*

7. If the **domain** of $f(x) = x^2$ is $\mathbb{R}$, it's **range** is $\mathbb{R}^+ \cup \{0\}$

8. If the **domain** of $f(x) = x^2$ is $\mathbb{N}$, it's **range** is the perfect squares (0, 1, 4, 9, 16, ...)

9. If the **domain** of $f(x) = x^3$ is $\mathbb{R}$, it's **range** is $\mathbb{R}$

10. If the **codomain** of $f(x) = \frac{1}{2^x}$ is $\mathbb{N}$ and f is total, $\mathbb{Z} \cap$ its **domain** is $\mathbb{Z}^- \cup \{0\}$

PROBLEM 3 *Provide example functions*

In each blank, define a total function $f : \mathbb{Z} \to \mathbb{Z}$

11. Give an example injective (1-to-1) and surjective (onto) function: $f(x) = x + 1$

12. Give an example injective (1-to-1) but not surjective (not onto) function: $f(x) = 2x$

13. Give an example non-injective (not 1-to-1) but surjective (onto) function: $f(x) = \left\lfloor \frac{x}{2} \right\rfloor$

14. Give an example neither injective (not 1-to-1) not surjective (not onto) function: $f(x) = x^2$

In each blank, define a function $f : \mathbb{N} \to \mathbb{N}$ or relation $R : \mathbb{N} \times \mathbb{N} \to \{\top, \bot\}$

15. Give an example function that is not total: $f(x)$ = $x - 1$

16. Give an example function that is total but not invertable: $f(x)$ = $(x-1)^2$

17. Give the relation corresponding to the function $f(x) = 3x$: $R(a, b)$: $a = 3b$

18. Give an example relation that is not a function: $R(x, y)$ = $x < y$

In each blank, define a function $f : \mathbb{R} \to \mathbb{R}$

Give an example function that is not total: $f(x)$ = $\sqrt{x}$

Give an example function that is total but not invertable: $f(x)$ = x^2

Give an example function that is invertable: $f(x)$ = x

See also §4 Problems 4.12–4.33