CS 2102 - DMT1 - Spring 2020 — Luther Tychonievich
Practice exercise in class friday april 1-, 2020

Practice 10

problem 1 *Convert to prose*

P: the set of all single-input functions
I: the set of all inputs
$C(p, i)$: p crashes when run on i

Convert the following to simple, readable English. Make sure your answer shows how the questions are different:

1. $\exists p \in P \,.\, \forall i \in I \,.\, C(p, i)$

2. $\exists i \in I \,.\, \forall p \in P \,.\, C(p, i)$

3. $\forall p \in P \,.\, \exists i \in I \,.\, C(p, i)$

4. $\forall i \in I \,.\, \exists p \in P \,.\, C(p, i)$

Convert the following to logic:

5. If a program crashes on any input, it crashes on more than one input.

6. No program crashes on every input.

PROBLEM 2 *Identify domain and range*

7. If the **domain** of $f(x) = x^2$ is $\mathbb{R}$, it's **range** is _____

8. If the **domain** of $f(x) = x^2$ is $\mathbb{N}$, it's **range** is _____

9. If the **domain** of $f(x) = x^3$ is $\mathbb{R}$, it's **range** is _____

10. If the **codomain** of $f(x) = \frac{1}{2^x}$ is $\mathbb{N}$ and f is total, $\mathbb{Z} \cap$ its **domain** is _____

PROBLEM 3 *Provide example functions*

In each blank, define a total function $f : \mathbb{Z} \to \mathbb{Z}$

11. Give an example injective (1-to-1) and surjective (onto) function: _____

12. Give an example injective (1-to-1) but not surjective (not onto) function: _____

13. Give an example non-injective (not 1-to-1) but surjective (onto) function: _____

14. Give an example neither injective (not 1-to-1) not surjective (not onto) function: _____

In each blank, define a function $f : \mathbb{N} \to \mathbb{N}$ or relation $R : \mathbb{N} \times \mathbb{N} \to \{\top, \bot\}$

15. Give an example function that is not total: $f(x)$ = _____

16. Give an example function that is total but not invertable: $f(x)$ = _____

17. Give the relation corresponding to the function $f(x) = 3x$: $R(a, b)$: _____

18. Give an example relation that is not a function: $R(x, y)$ = _____

In each blank, define a function $f : \mathbb{R} \to \mathbb{R}$

Give an example function that is not total: $f(x)$ = _____

Give an example function that is total but not invertable: $f(x)$ = _____

Give an example function that is invertable: $f(x)$ = _____

See also §4 Problems 4.12–4.33