Name: ______________________________ CompID: ______________

CS 2102 - DMT1 - Spring 2020 — Luther Tychonievich
Administered in class friday february 14, 2020

Quiz 04

problem 1 *Symbolizing*

1. Provide a logic translation for "I'm known by someone so famous that everyone knows them."
   domain: people

$K(x, y)$: x knows y
m: Me

$$\exists x \,.\, K(x, m) \land \forall y \,.\, K(y, x)$$

problem 2 *Prosify*

Convert the following proof into a prose proof.

2. Definition: A positive integer is **abundant** if it is smaller than the sum of its factors.
Theorem: There is at least one abundant integer.
Formalism: $\exists x \,.\, A(x)$ (domain: positive integers; $A(x)$: x is abundant)
Proof outline: 12 has factors $1, 2, 3, 4, 6$; $1 + 2 + 3 + 4 + 6 = 16 \vdash A(12)$; $A(12) \vdash \exists x \,.\, A(x)$

Proof.

Consider the number 12. The factors of 12 are $1, 2, 3, 4$, and 6. The sum of those factors is 16, which is greater than 12.
Since the sum of the factors of 12 is greater than 12, 12 is abundant, meaning there is at least one abundant number.

PROBLEM 3 *Complete*

Fill in the blanks to complete the following proof by cases that $P \to (P \vee Q)$ is a tautology.

Proof. 3. Either Q ______ is true or it is false.

Case 1: Q ______ **is true** The expression $P \to (P \vee Q)$ in this case

4. can be simplified to $P \to \top$,

which is equivalent to $\top$.

Case 2: Q ______ **is false** The expression $P \to (P \vee Q)$ in this case

5. can be simplified to $P \to P$,

which is equivalent to $\top$.

Since $P \to (P \vee Q)$ is true in both cases, it is true in general, meaning it is a tautology. □

Symbols

Concept	Java/C	Python	This class	Bitwise	Other
true	`true`	`True`	$\top$ or 1	`-1`	T, tautology
false	`false`	`False`	$\bot$ or 0	`0`	F, contradiction
not P	`!p`	`not p`	$\neg P$ or $\overline{P}$	`~p`	
P and Q	`p && q`	`p and q`	$P \wedge Q$	`p & q`	PQ, $P \cdot Q$
P or Q	`p || q`	`p or q`	$P \vee Q$	`p | q`	$P + Q$
P xor Q	`p != q`	`p != q`	$P \oplus Q$	`p ^ q`	$P \veebar Q$
P implies Q			$P \to Q$		$P \supset Q$, $P \Rightarrow Q$
P iff Q	`p == q`	`p == q`	$P \leftrightarrow Q$		$P \Leftrightarrow Q$, P xnor Q

Axioms: Equivalence rules

- associativity and commutativity of $\wedge$, $\vee$, and $\oplus$; commutativity of $\leftrightarrow$
- double negation: $\neg\neg P \equiv P$
- simplification: $P \wedge \bot \equiv P \wedge \neg P \equiv \bot$, $P \vee \top \equiv P \vee \neg P \equiv \top$, and $P \wedge \top \equiv P \vee \bot \equiv P \wedge P \equiv P \vee P \equiv P$
- distribution: $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$ and $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$
- De Morgan: $\neg(A \wedge B) \equiv (\neg A) \vee (\neg B)$ and $\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)$
- definitions: $A \to B \equiv (\neg A) \vee B$, $(A \leftrightarrow B) \equiv (A \to B) \wedge (B \to A)$ and $(A \oplus B) \equiv (A \vee B) \wedge \neg(A \wedge B)$