Name: ______________________________ CompID: ______________

CS 2102 - DMT1 - Spring 2020 — Luther Tychonievich
Administered in class friday february 14, 2020

Quiz 04

problem 1 *Symbolizing*

1. Provide a logic translation for "I'm known by someone so famous that everyone knows them."
   domain: people

problem 2 *Prosify*

Convert the following proof into a prose proof.

2. Definition: A positive integer is **abundant** if it is smaller than the sum of its factors.
Theorem: There is at least one abundant integer.
Formalism: $\exists x . A(x)$ (domain: positive integers; $A(x)$: x is abundant)
Proof outline: 12 has factors $1, 2, 3, 4, 6$; $1 + 2 + 3 + 4 + 6 = 16 \vdash A(12)$; $A(12) \vdash \exists x . A(x)$

Proof.

PROBLEM 3 *Complete*

Fill in the blanks to complete the following proof by cases that $\boxed{P \rightarrow (P \vee Q)}$ is a tautology.

Proof. 3. Either ________ is true or it is false.

Case 1: ________ is true The expression $P \rightarrow (P \vee Q)$ in this case

4.

which is equivalent to $\top$.

Case 2: ________ is false The expression $P \rightarrow (P \vee Q)$ in this case

5.

which is equivalent to $\top$.

Since $P \rightarrow (P \vee Q)$ is true in both cases, it is true in general, meaning it is a tautology. □

Symbols

Concept	Java/C	Python	This class	Bitwise	Other
true	`true`	`True`	$\top$ or 1	`-1`	T, tautology
false	`false`	`False`	$\bot$ or 0	`0`	F, contradiction
not P	`!p`	`not p`	$\neg P$ or $\overline{P}$	`~p`	
P and Q	`p && q`	`p and q`	$P \wedge Q$	`p & q`	PQ, $P \cdot Q$
P or Q	`p || q`	`p or q`	$P \vee Q$	`p | q`	$P + Q$
P xor Q	`p != q`	`p != q`	$P \oplus Q$	`p ^ q`	$P \veebar Q$
P implies Q			$P \rightarrow Q$		$P \supset Q$, $P \Rightarrow Q$
P iff Q	`p == q`	`p == q`	$P \leftrightarrow Q$		$P \Leftrightarrow Q$, P xnor Q

Axioms: Equivalence rules

- associativity and commutativity of $\wedge$, $\vee$, and $\oplus$; commutativity of $\leftrightarrow$
- double negation: $\boxed{\neg\neg P \equiv P}$
- simplification: $\boxed{P \wedge \bot \equiv P \wedge \neg P \equiv \bot}$, $\boxed{P \vee \top \equiv P \vee \neg P \equiv \top}$, and $\boxed{P \wedge \top \equiv P \vee \bot \equiv P \wedge P \equiv P \vee P \equiv P}$
- distribution: $\boxed{A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)}$ and $\boxed{A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)}$
- De Morgan: $\boxed{\neg(A \wedge B) \equiv (\neg A) \vee (\neg B)}$ and $\boxed{\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)}$
- definitions: $\boxed{A \rightarrow B \equiv (\neg A) \vee B}$, $\boxed{(A \leftrightarrow B) \equiv (A \rightarrow B) \wedge (B \rightarrow A)}$ and $\boxed{(A \oplus B) \equiv (A \vee B) \wedge \neg(A \wedge B)}$