Name: ______________________________ CompID: ______________

CS 2102 - DMT1 - Spring 2020 — Luther Tychonievich
Administered in class friday february 28, 2020

Quiz 06

problem 1 *Convert to prose*

Convert the following symbolic proof that $\texttt{f(x)} = (\texttt{x})(\texttt{x}+1)$ to prose.

```
let f(x) be computed as
    if x <= 0 then return 0
    else return 2*x + f(x-1)
```

Symbolic Proof.

1		$f(0) = 0 = (0)(0+1)$	definition
2	2	$f(x-1) = (x-1)(x)$	assumption
	3	$f(x) = 2x + f(x-1)$	definition
	4	$f(x) = 2x + (x-1)(x)$	combine line 2 and 3
	5	$f(x) = 2x + (x^2 - x)$	algebra on line 4
	6	$f(x) = x^2 + x$	algebra on line 5
	7	$f(x) = (x)(x+1)$	simplify line 6
3		$\forall x \geq 0 \,.\, f(x) = (x)(x+1)$	principle of induction on lines 1 and 2

Proof.

We proceed by induction on x.

Base case: Assume $\texttt{x} = 0$. Then $\texttt{f(x)} = 0$, which is equal to $(x)(x+1)$ when $x = 0$.

Inductive step: Assume that $\texttt{x} > 0$ and that `f(x-1) = (x-1)*(x)`. Then `f(x) = 2*x + f(x-1)` which, by the assumption, is `2*x + (x-1)*x`; rearranging, that becomes `x*x + x`, which is $(x)(x+1)$.

By the principle of induction, it follows that `f(x)` always returns $(\texttt{x})(\texttt{x}+1)$. □

PROBLEM 2 *Code termination*

Prove by induction that each of the following function terminates given any natural number argument.

2.
```
let f(x) be computed as
    if x == 0 then return 1
    otherwise return 2*f(x-1)
```

Proof.

We proceed by induction on `x`.

Base case: Assume $x = 0$. Then the function terminates immediately by taking the first branch of the `if` statement.

Inductive step: Assume that $x > 0$ and that `f(x-1)` terminates. Then the function takes the second branch of the `if` statement and terminates after invoking `f(x-1)` and performing one multiplication.

By the principle of induction, it follows that `f(x)` terminates for all integer `x`. □