Name: ______________________________ CompID: ______________

CS 2102 - DMT1 - FALL 2020 — LUTHER TYCHONIEVICH
ADMINISTERED IN CLASS FRIDAY APRIL 3, 2020

QUIZ 09

PROBLEM 1 *Counting*

Number and write your answers to the following. You may use factorial, choose, and arithmetic notation, but may not use ellipses. For example, the following are all OK: $\boxed{120}$, $\boxed{5!}$, $\boxed{\frac{5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1}}$, $\boxed{\binom{5}{3}}$; however, the following is *not* OK: $\boxed{10+9+\cdots+2+1}$.

There are 10 digits: 0 through 9.

There are 26 letters: 'a' through 'z'.

1. $2^{10} = 1024$ How many sets of digits are there?

2. $\binom{10}{6} = \frac{10\cdot9\cdot8\cdot7}{4\cdot3\cdot2\cdot1} = 5040$ How many 6-element sets of digits are there?

3. $26\cdot(10+26)^5 = 1572120576$ How many 6-character identifiers can be made with a leading letter following by 5 digits or letters (e.g. `"cs2102"` or `"dmt1ok"` but not `"2012ok"`)?

4. $\frac{6}{6^3} = \frac{1}{6^2} = \frac{1}{36}$ I roll three fair six-sided dice. What is the probability I roll the same number on all three?

5. $\frac{6!}{2!} - 1 = 359$ Two strings are anagrams if they are distinct but can have their characters rearranged to be the same. How many anagrams of `"cs2102"` are there?

PROBLEM 2 *Proofs*

Prove by induction that

$$\forall n \in \mathbb{N} \,.\, \left(\sum_{x=0}^{n}(2x+1)\right) = (n+1)^2$$

Proof. We proceed by induction.

Base Case When $n = 0$ we have $\sum_{x=0}^{0}(2x+1) = 1 = (2 \cdot 0 + 1)^2$, so the theorem holds for $n = 0$.

Inductive step Assume the theorem holds for some $n = k$; that is, $\sum_{x=0}^{k}(2x+1) = (k+1)^2$. Consider the sum evaluated at $k+1$:

$$\begin{aligned}
\sum_{x=0}^{k+1}(2x+1) &= \big(2(k+1)+1\big) + \sum_{k=0}^{k}(2x+1) \\
&= (2k+3) + (k+1)^2 \\
&= 2k+3+k^2+2k+1 \\
&= k^2+4k+4 \\
&= (k+2)^2 \\
&= \big((k+1)+1\big)^2
\end{aligned}$$

which means the theorem holds at $k+1$ as well.

By the principle of induction, the theorem holds for all $n \in \mathbb{N}$. $\square$