CS2120
Discrete Math
Jan 24th

Elizabeth Orrico

Sets

1.) Discord!
2.) About Quizzes
3.) Set Definition
4.) $\in$
5.) $\subseteq$, $\subset$, $\supseteq$, $\supset$
6.) $\cup$, $\cap$, $\setminus$
7.) Set Cover

Python Sets vs Lists

Ask Question

Asked 10 years, 3 months ago Active 16 days ago Viewed 149k times

188

In Python, which data structure is more efficient/speedy? Assuming that order is not important to me and I would be checking for duplicates anyway, is a Python set slower than a Python list?

python list performance data-structures set

54

share improve this question follow

edited Aug 12 '19 at 5:59
user11768920

asked May 14 '10 at 0:55
Mantas Vidutis
14.5k ●20 ●72 ●90

add a comment

9 Answers

Active Oldest Votes

234

It depends on what you are intending to do with it.

Sets are significantly faster when it comes to determining if an object is present in the set (as in `x in s`), but are slower than lists when it comes to iterating over their contents.

You can use the timeit module to see which is faster for your situation.

share improve this answer follow

edited Sep 29 '16 at 10:25
smerlin
5,780 ●3 ●29 ●51

answered May 14 '10 at 1:04
Michael Aaron Safyan
84.7k ●13 ●126 ●192

https://www.cs.virginia.edu/~emo7bf/cs2120/sets.html

Defining Sets

A **set** is a structure that contains elements.

Defining Sets

A **set** is a structure that contains elements.

A **member** or **element** is something inside the set.

Defining Sets

A set is written with curly braces, its members separated by commas.

Examples: {1, 3} or {dog, cat, mouse}

Defining Sets

A member of a set has no other properties by virtue of being in a set.

How is this different from lists (in coding)?

Defining Sets

A member of a set has no other properties by virtue of being in a set.

Remember! No order, no duplicates.

Defining Sets

A member of a set has no other properties by virtue of being in a set.

{1, 3, 4, 1} doesn’t make sense.

{1, 2, 3} and {2, 3, 1} are the same set.

Sets can be members of other sets!

and are frequently

$\{\{1, 2\}, \{2, 1\}\}$

Sets can be members of other sets!

and are frequently

~~$\{\{1, 2\}, \{2, 1\}\}$~~

Sets can be members of other sets!

and are frequently

$$\{\{1, 2\}, 1\}$$

Sets can be members of other sets!

and are frequently

$$\{\{1, 2\}, 1\}$$

$$\{\{1, 2\}, \{1, 2, 3\}\}$$

THE Empty Set

A set with no members is empty.

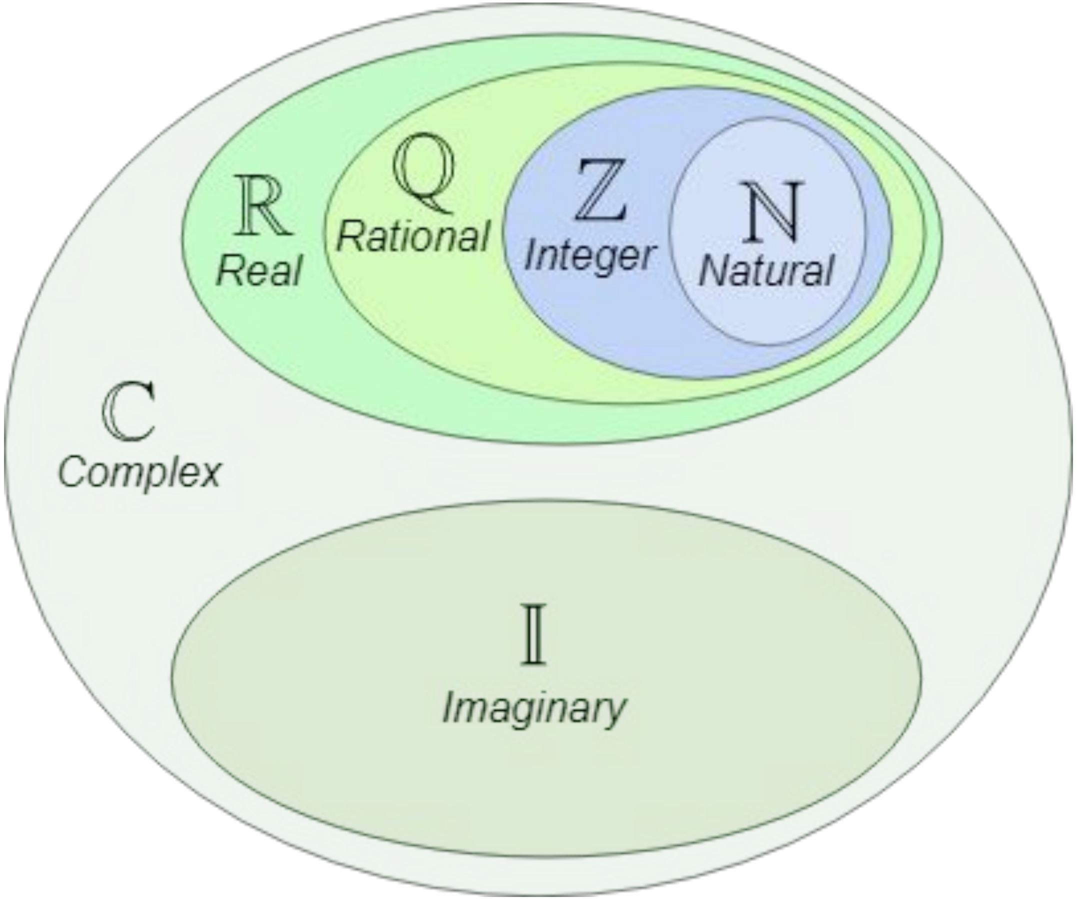
The empty set is expressed as {} or $\varnothing$

Cardinality

Q: Compute each cardinality.

1. $|\{1, -13, 4, -13, 1\}|$
2. $|\{3, \{1,2,3,4\}, \varnothing\}|$
3. $|\{\}|$
4. $|\{ \{\}, \{\{\}\}, \{\{\{\}\}\} \}|$

7.)



https://www.mathsisfun.com/sets/number-types.html

$\in$

"Element of"

$$\in$$

Python: “in”

Java: “contains”

Evaluates to true or false

Examples

$2 \in \{1, 2\}$ = ________

$3 \in \{1, 2\}$ = ________

Examples

$2 \in \{1, 2\}$ = ________

$3 \in \{1, 2\}$ = ________

$3 \notin \{1, 2\}$ = ________

Question

$\{2\} \in \{1, 2\}$ = ________

Question

$\{2\} \in \{ 1, \{2\} \} \quad = \quad$ __________

2-min Talk

Evaluate true or false with your partners. *For each problem,* **have a different person start speaking/explaining first**

$\{2\} \in \{ \{1, 2\} \}$ = __________

$\{2\} \in \{ \{2\} \}$ = __________

$\{\{2\}\} \in \{ \{\{2\}\} \}$ = __________

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Jan 26th

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More Operators

∈ checks membership of an element

⊆, ⊂, ⊇, ⊃ compares two sets

⊆	subset
⊇	superset
⊂	proper subset
⊃	proper superset

More Operators

Set A is a ***subset*** of set B

$A \subseteq B$

If & only if **all elements of A are also in B**

More Operators

Set A is a ***proper subset*** of set B

$A \subset B$

If & only if **$A \subseteq B$ and $A \neq B$**

More Operators

Set A is a ***proper subset*** of set B

$A \subset B$

If & only if **$A \subseteq B$ and $A \neq B$**

*What are the **<u>consequences</u>*** of this definition?

Turn n' Talk -- 2 min

Given the three sets: **P = {1, 2, 3}, Q = {1, 3}, R = {1, 3, 4}**

Determine which (if any) symbol can be filled in each blank so the expression evaluates to true:

$\subseteq$ subset

$\supseteq$ superset

$\subset$ proper subset

$\supset$ proper superset

P ___ Q = T

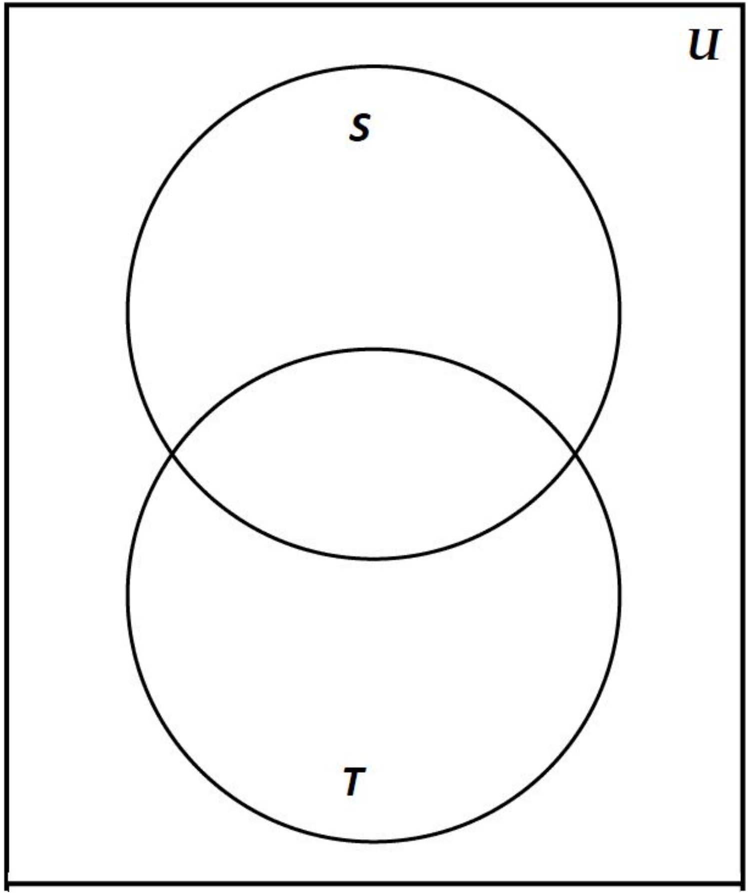
P ___ R = T

Q ___ R = T

P ___ P = T

In mathematics, the **intersection** of two sets S and T, denoted by S ∩ T, is the set containing all elements of S that also belong to T (or equivalently, all elements of T that also belong to S)

∩



DEFINITIONS: the **intersection** of two sets S and T, denoted by S ∩ T, is the set containing all elements of S that also belong to T (or equivalently, all elements of T that also belong to S)

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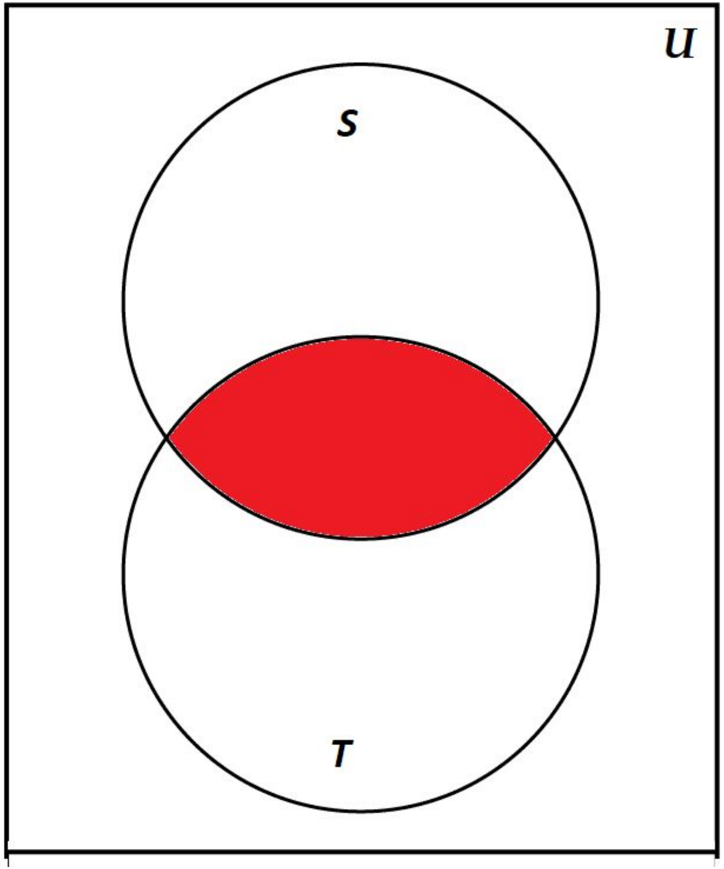
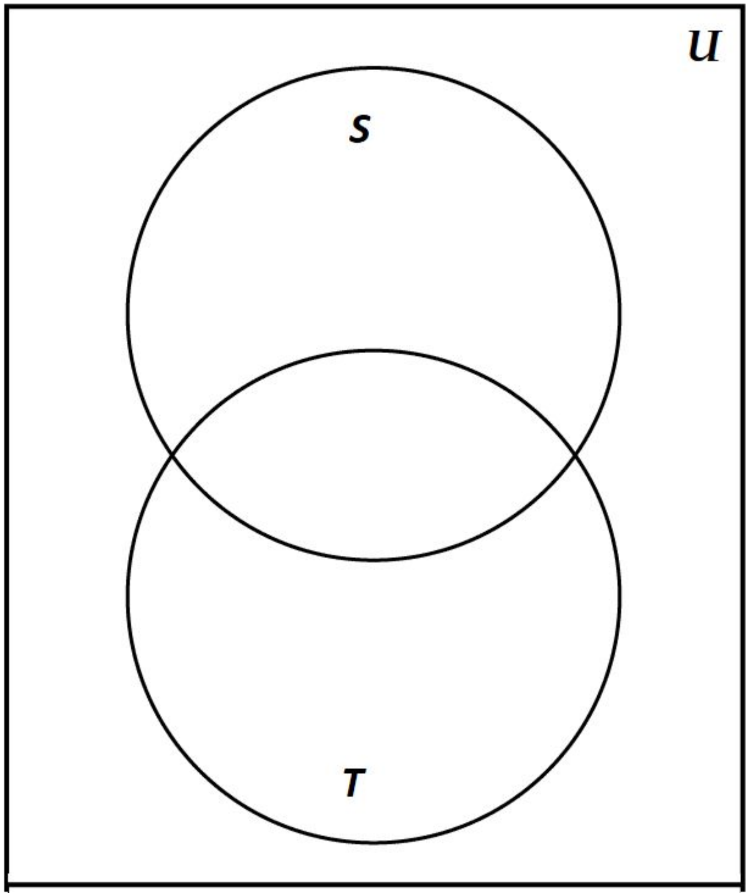


In mathematics, the **union** of two sets S and T, denoted by S ∪ T, is the set containing all the elements of S *and* also all the elements T (or equivalently, everything in either S or T or both)

∪



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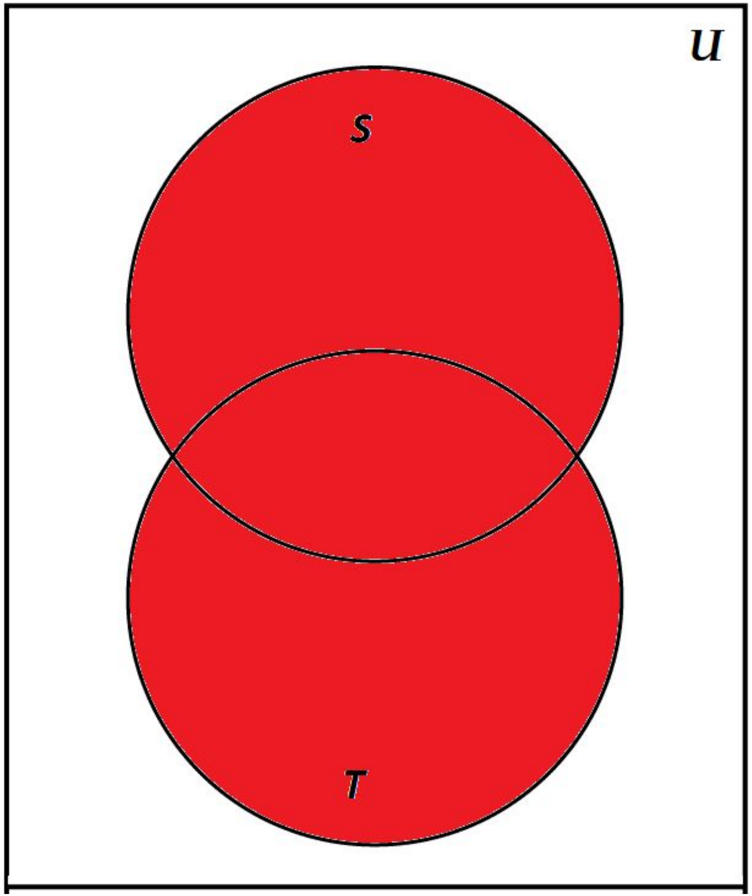
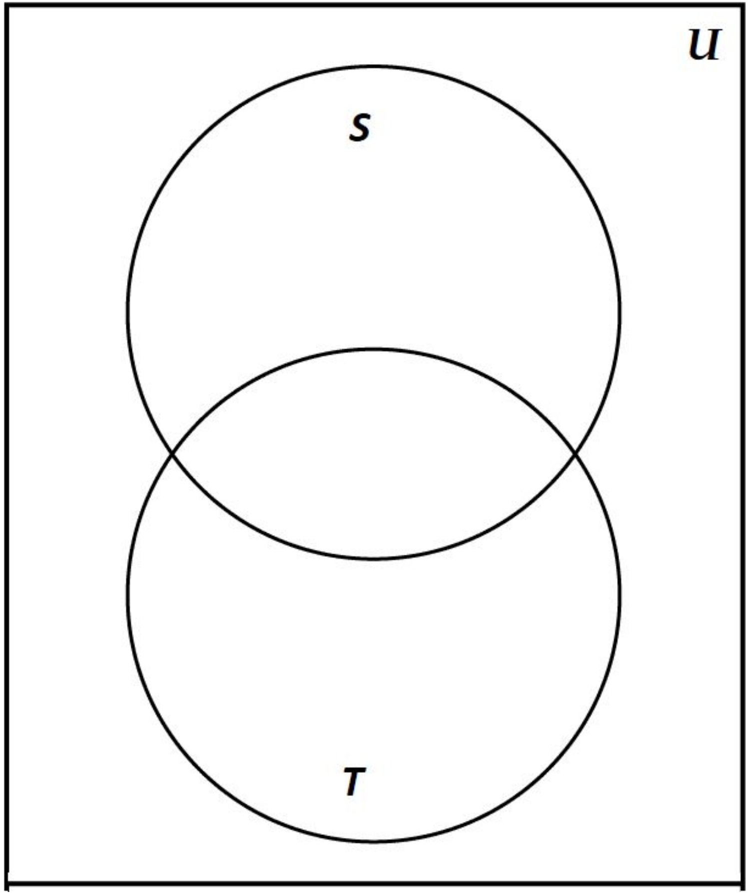


Difference $S \setminus T$: the elements that belong to S but not to T.

S\T



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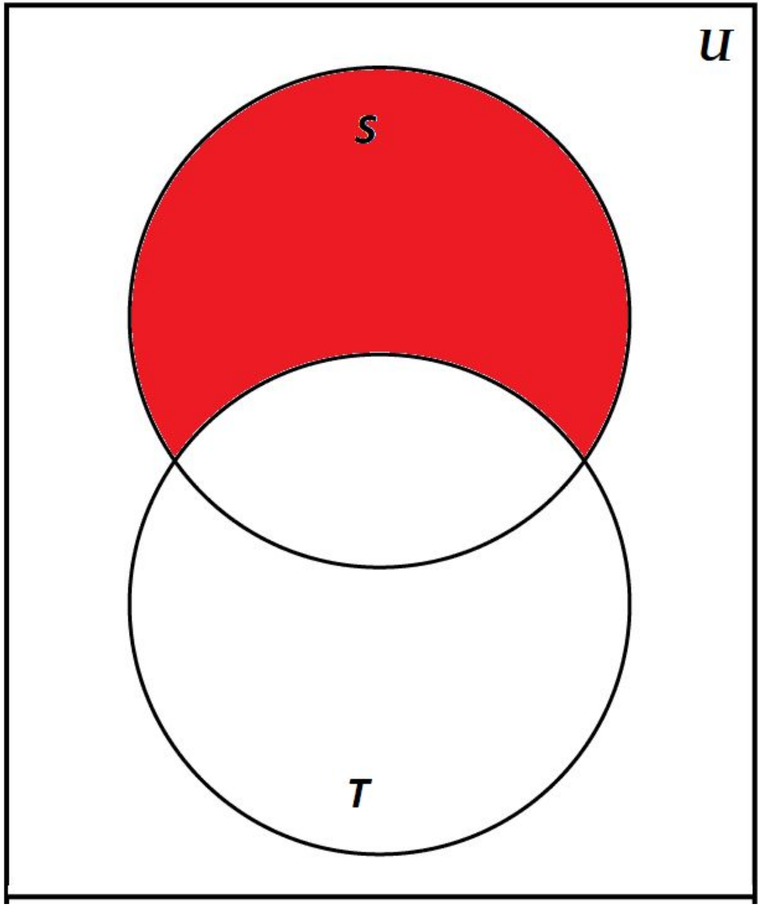
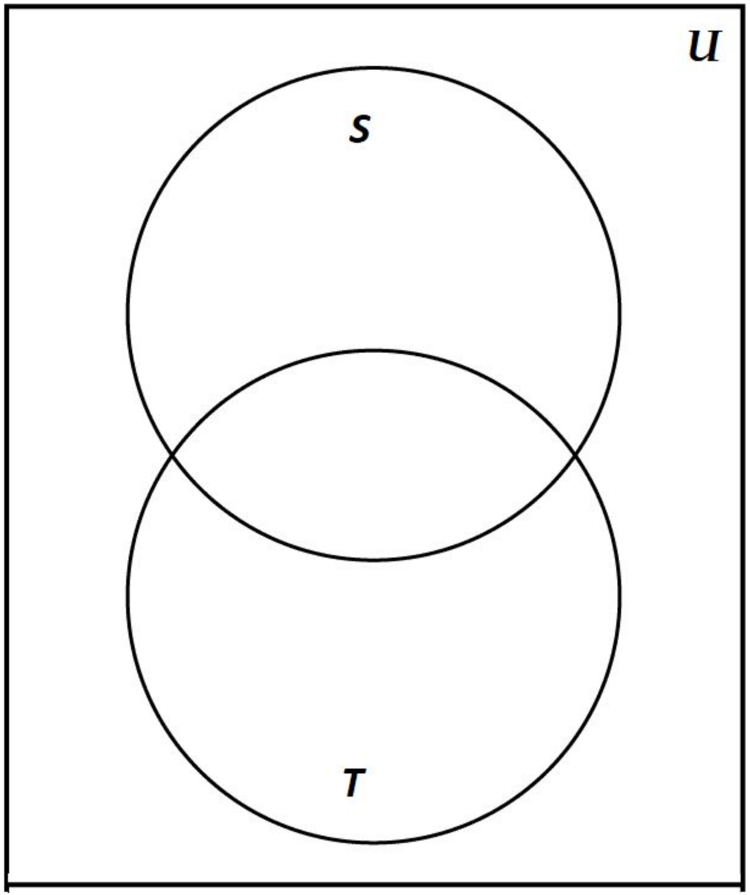


Complement $\overline{S}$: elements (of the universe) that don't belong to S.

$\overline{S}$



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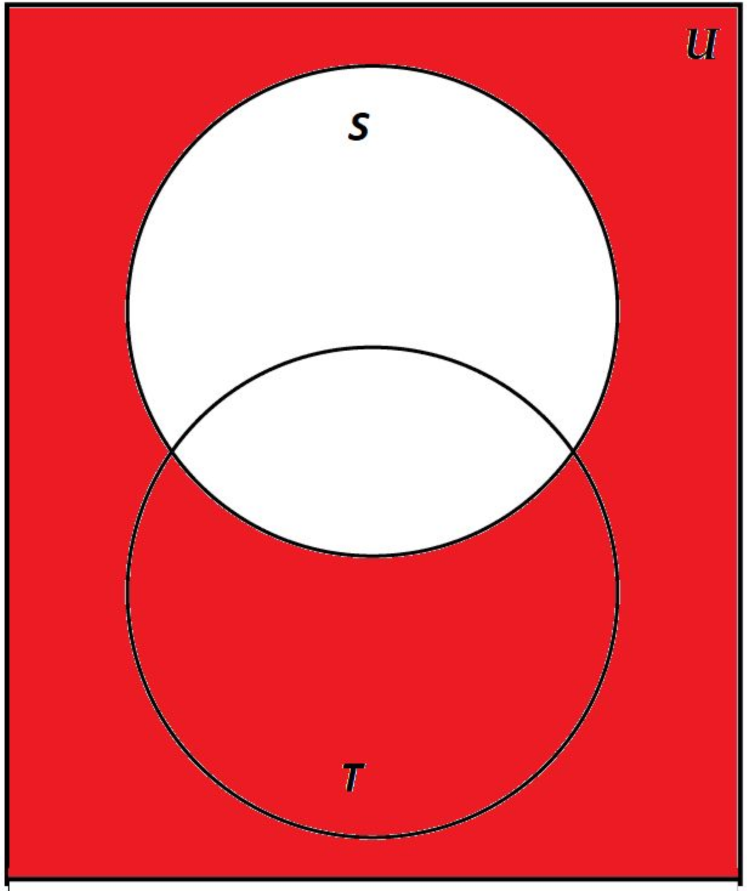

Properties Of Set Union

- $A \cup \varnothing =$
- $A \cup \boldsymbol{U} =$
- $A \cup A =$

Properties Of Set Union

- $A \cup \varnothing = A$ Identity law
- $A \cup \boldsymbol{U} =$
- $A \cup A =$

Properties Of Set Union

- $A \cup \varnothing = A$ Identity law
- $A \cup \boldsymbol{U} = \boldsymbol{U}$ Domination law
- $A \cup A =$

Properties Of Set Union

- A U $\varnothing$ = A — Identity law
- A U $\boldsymbol{U}$ = $\boldsymbol{U}$ — Domination law
- A U A = A — Idempotent law

Properties Of Set Union

- $A \cup \varnothing = A$ — Identity law
- $A \cup \boldsymbol{U} = \boldsymbol{U}$ — Domination law
- $A \cup A = A$ — Idempotent law
- $A \cup B = B \cup A$ — Commutative law

Properties Of Set Intersection

- $A \cap \boldsymbol{U} =$
- $A \cap \varnothing =$
- $A \cap A =$

Properties Of Set Intersection

- $A \cap \boldsymbol{U} = A$ Identity law
- $A \cap \varnothing =$
- $A \cap A =$

Properties Of Set Intersection

- $A \cap \boldsymbol{U} = A$ Identity law
- $A \cap \varnothing = \varnothing$ Domination law
- $A \cap A =$

Properties Of Set Intersection

- $A \cap \boldsymbol{U} = A$ — Identity law
- $A \cap \varnothing = \varnothing$ — Domination law
- $A \cap A = A$ — Idempotent law

Properties Of Set Intersection

$A \cap \boldsymbol{U} = A$	Identity law
$A \cap \varnothing = \varnothing$	Domination law
$A \cap A = A$	Idempotent law
$A \cap B = B \cap A$	Commutative law

Your Turn! Draw a Venn Diagram with 3 intersecting sets.

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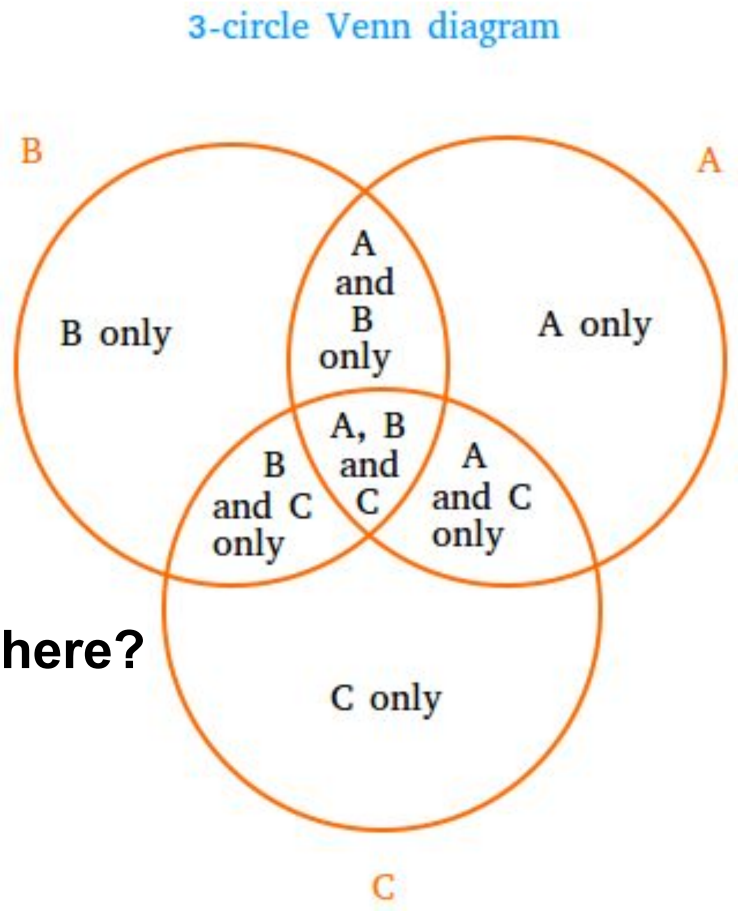
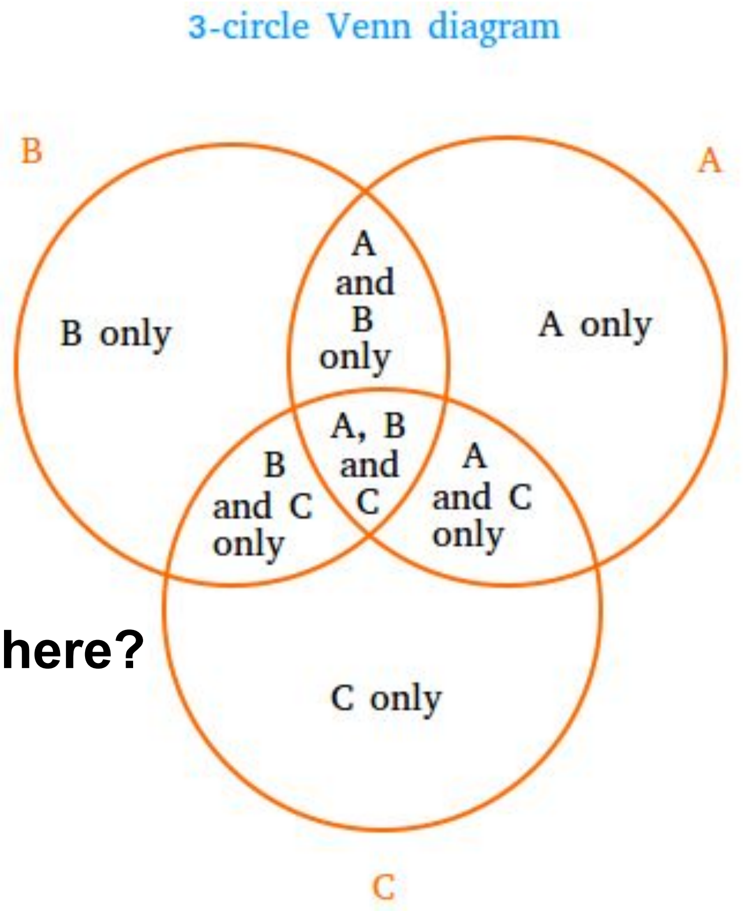


How many regions are there?

Your Turn! Draw a Venn Diagram with 3 intersecting sets.



How many regions are there?

Does this make sense?

Your Turn! Draw a Venn Diagram with 4 intersecting sets.

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Hint #1: It is possible

Your Turn! Draw a Venn Diagram with 4 intersecting sets.

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Hint #2: The sets cannot be represented by perfect circles.

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Hint #3: The regions are ellipses (ovals).

Your Turn! Draw a Venn Diagram with 4 intersecting sets.

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Hint #3: The regions are ellipses (ovals).

How many regions are there?

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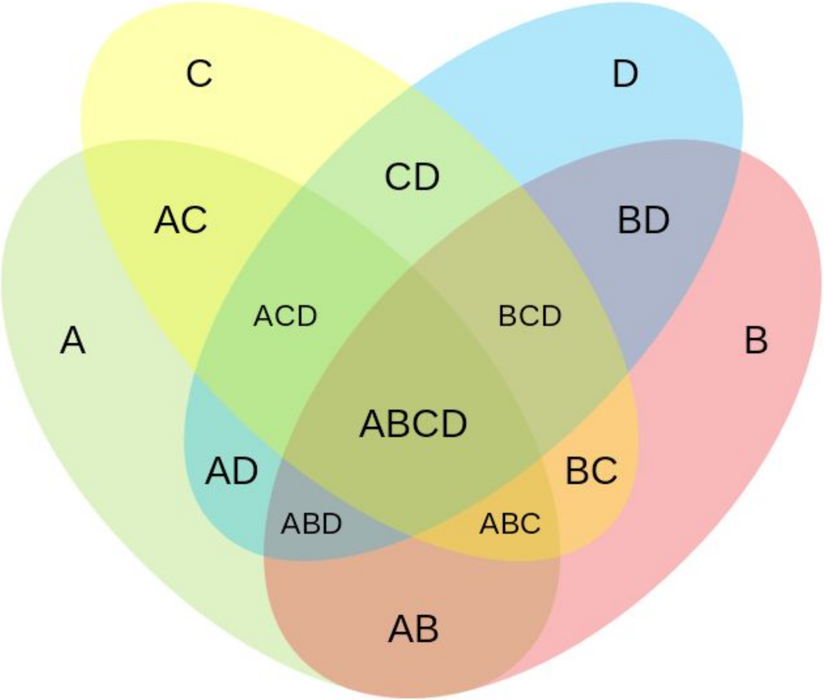

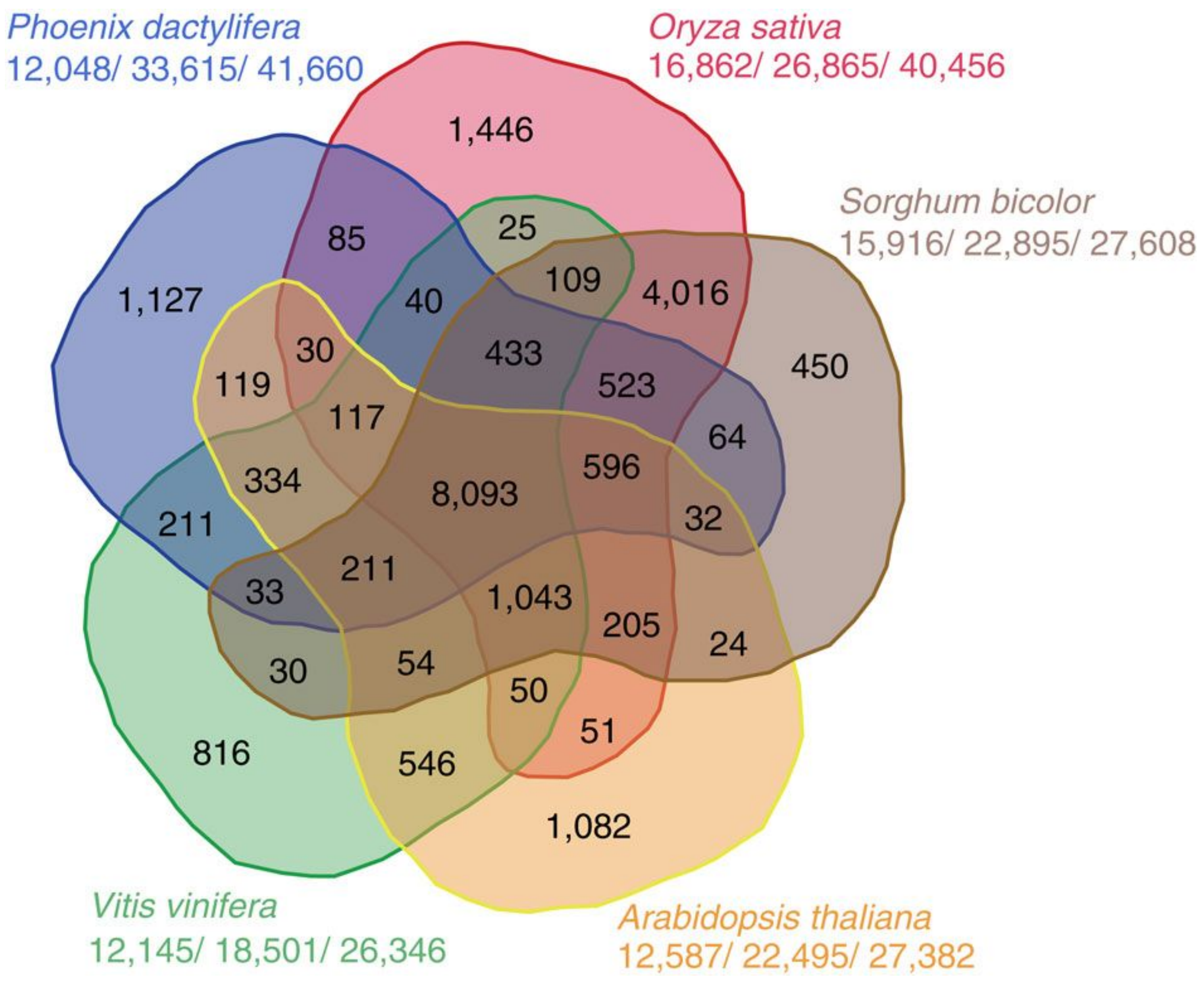
Phoenix dactylifera
12,048/ 33,615/ 41,660
Oryza sativa
16,862/ 26,865/ 40,456
Sorghum bicolor
15,916/ 22,895/ 27,608
Vitis vinifera
12,145/ 18,501/ 26,346
Arabidopsis thaliana
12,587/ 22,495/ 27,382
1,446
85
25
1,127
40
109
4,016
30
433
119
523
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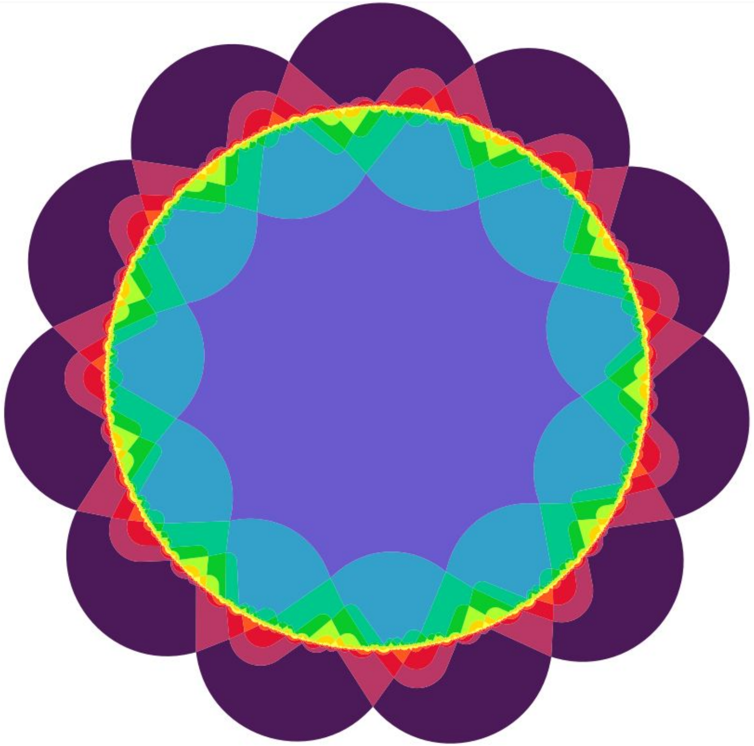


Fig. 3: Newroz, the first simple symmetric 11-Venn diagram.

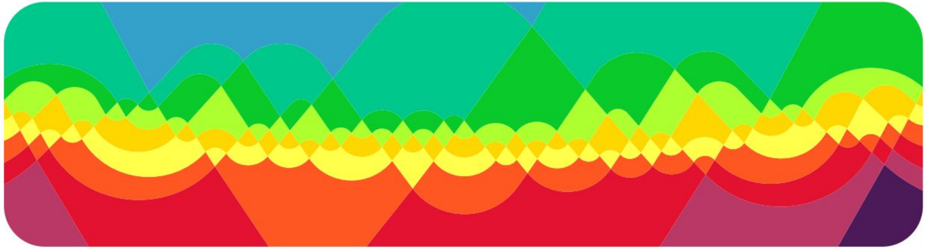


Fig. 4: A blow-up of part of Newroz.

https://arxiv.org/pdf/1207.6452.pdf

Sidebar: Set Cover Problem

A very famous and useful problem in combinatorics and CS! One of the original problems to be proven **NP-Complete.**

One Example: Given a **"universe"** U (big set with everything else in the problem inside) and a set of sets, S

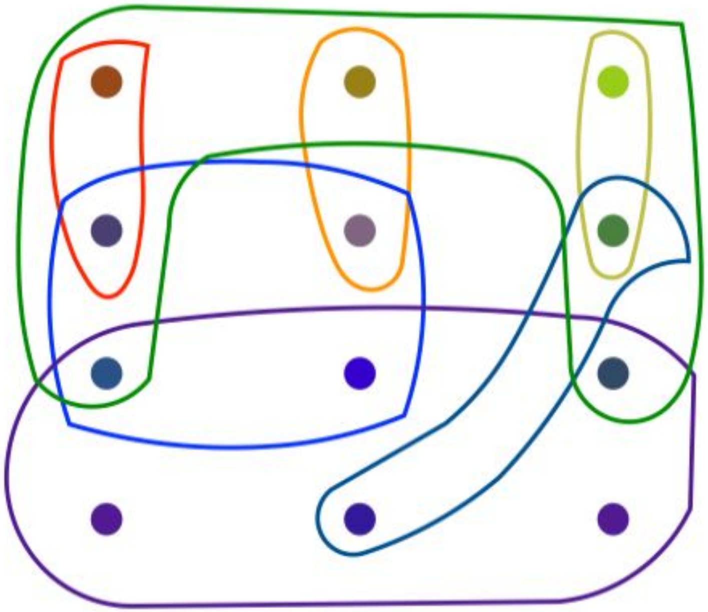
$$U = \{1, 2, 3, 4, 5\}$$

$$S = \{ \{1, 2, 3\}, \{2, 5, 1\}, \{3, 4, 1\}, \{3, 4, 5\} \}$$

What is the *minimum number* of sets in S needed to cover everything in U?

Sidebar: Set Cover Problem

Your turn!



Input

???

Output

Supose $A \subset B$ and $B \subseteq C$ are true, and $B \subseteq D$ and $D \subseteq B$ are both false.

For each of the following, decide if it

- must, could, or can't be empty
- how it must relate ($\subseteq$, $\subset$, $\supseteq$, $\supset$, $=$) to the four named sets (if any)

a. $A \cap B$
b. $A \cup B$
c. $B \cap C$
d. $B \cup C$
e. $B \cap D$
f. $B \cup D$
g. $C \cap D$
h. $C \cup D$
i. $A \cap C$
j. $A \cup C$
k. $A \cap D$
l. $A \cup D$

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Sets and Sequences, Continued

1.) Sequences vs Sets
2.) Cartesian Product
3.) Set-builder notation
4.) Set Operator Review

High Level: Sets vs Sequences

Both can:
-Contain anything
-Can have a sequence of sequences, set of sets, sequence of sets, etc
-Cannot be modified

Sets:
-no duplicates
-no order
-has cardinality

Sequences:
-can have duplicates
-has order
-has length

Lists, Arrays, Ordered pairs, Tuples, etc!

Cartesian Product of Sets

Ordered Pair: An ordered pair is a **sequence with 2 elements**. It is a pair of objects where one element is designated first and the other element is designated second, denoted (a, b).

Cartesian Product: The Cartesian product of two sets A and B, denoted A × B, is the set of all possible ordered pairs where the elements of A are first and the elements of B are second.

Cartesian Product of Sets

Ex: $\{1, 2\} \times \{3, 4, 5\}$

$$= \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

Cartesian Product of Sets

Ex: $\{1, 2\} \times \{3, 4, 5\}$

$$= \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

Cartesian Product of Sets

Ex: $\{1, 2\} \times \{3, 4, 5\}$

$$= \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

Cartesian Product of Sets

Ex: $\{1, 2\} \times \{3, 4, 5\}$

$$= \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{2, 3\}$?

Cartesian Product of Sets

Your: $|\{1, 2, 3\} \times \{3, 4, 5\}| =$

Cartesian Product of Sets

Ex: $|\{1, 2, 3\} \times \{3, 4, 5\}|$

$$= |\{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}|$$

Cartesian Product of Sets

Ex: $|\{1, 2\} \times \{3, 4, 5\}|$

$$= |\{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}|$$

$$= 6$$

Cartesian Product of Sets

$$|\{\ \{\}\ \} \times \{1, 2, 3\}|$$

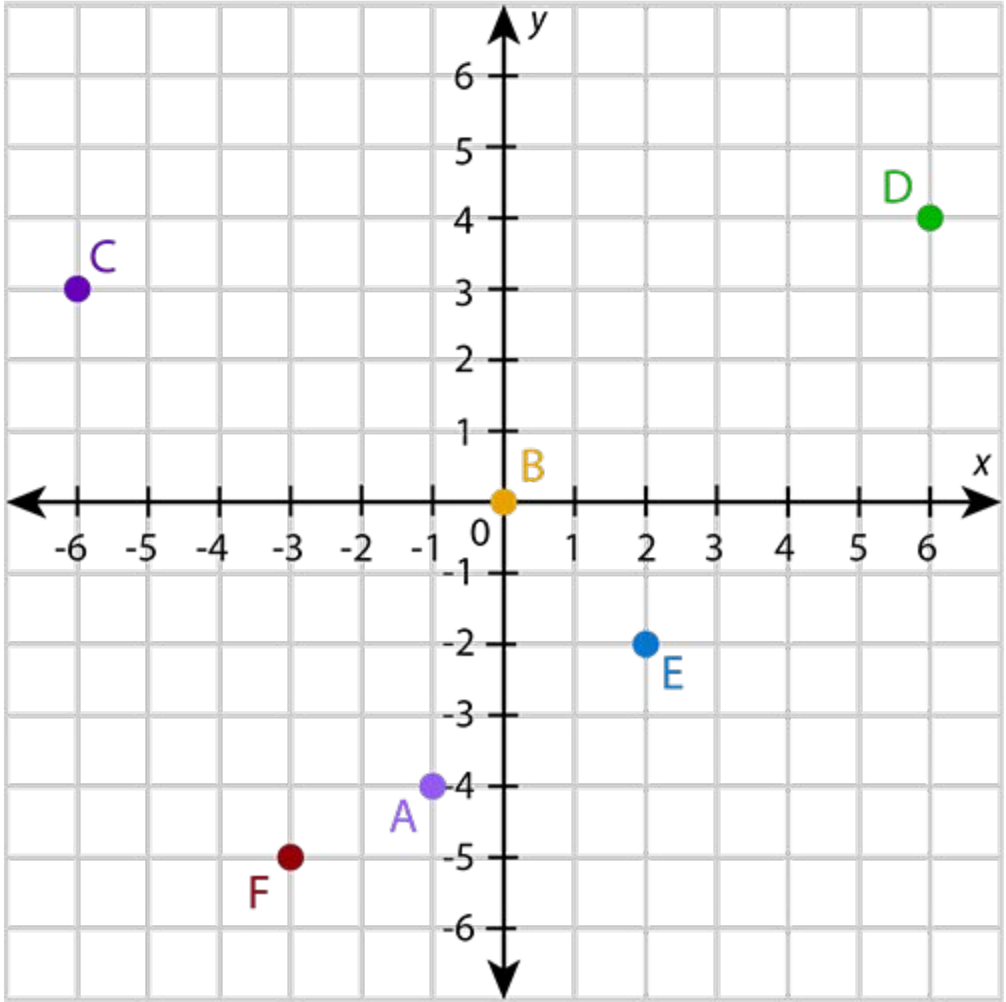
$$= ??????$$

Cartesian Product of Sets

$$|\{\ \{\}\ \} \times \{1, 2, 3\}|$$

$$= \{\ (\ \{\}, 1), (\{\}, 2), (\{\}, 3)\ \}$$

What is $\{-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}^2$?


y
x
6
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-1
-2
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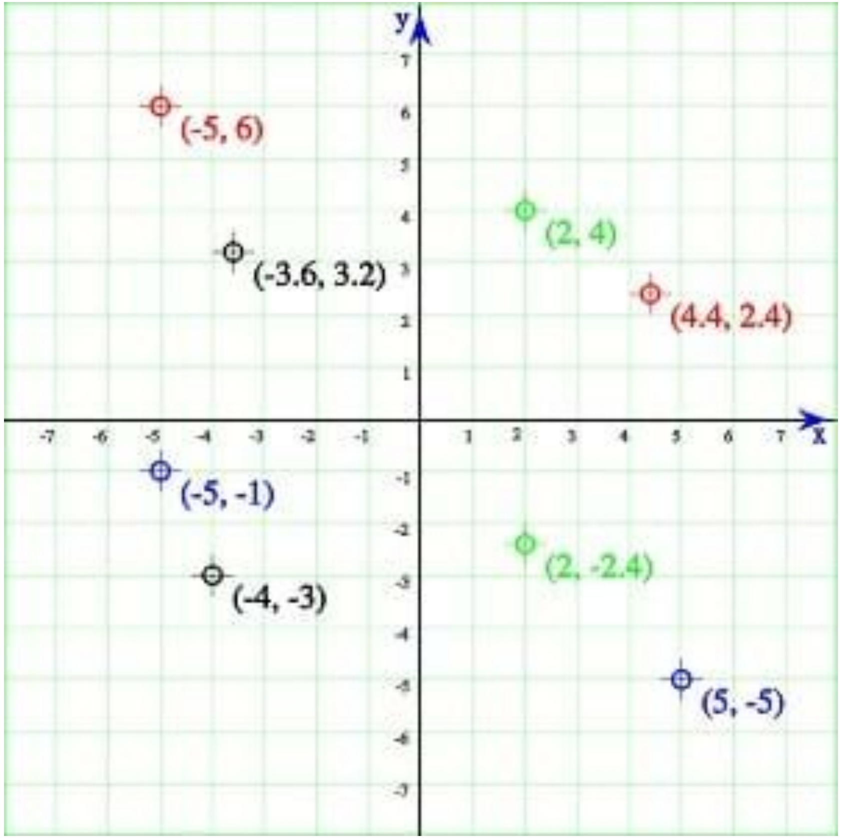
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6
A
B
C
D
E
F

Cartesian Product of Sets



$\mathbb{R} \times \mathbb{R}$: The coordinate plane

Ways to describe Sets

- Listing out the elements of a set works well for sets that are small and finite. What about larger sets?
- Set Builder Notation!

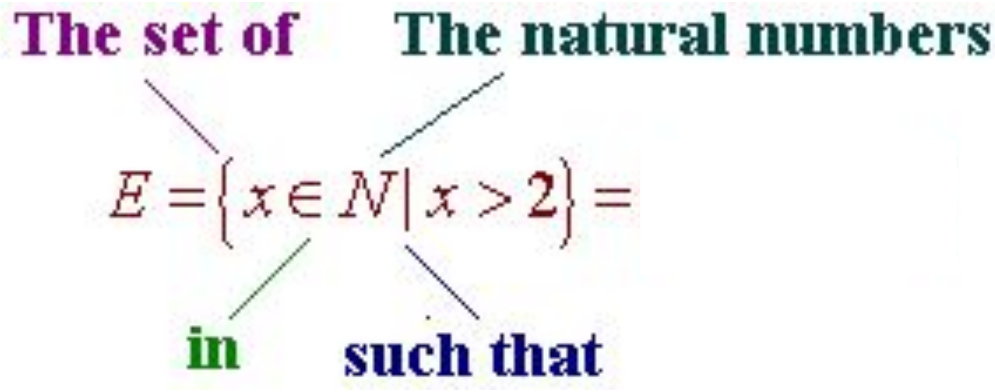
$$S = \{x \in A | x \text{ is blue}\}$$

The set of all x in A

Vertical Bar is read "such that"

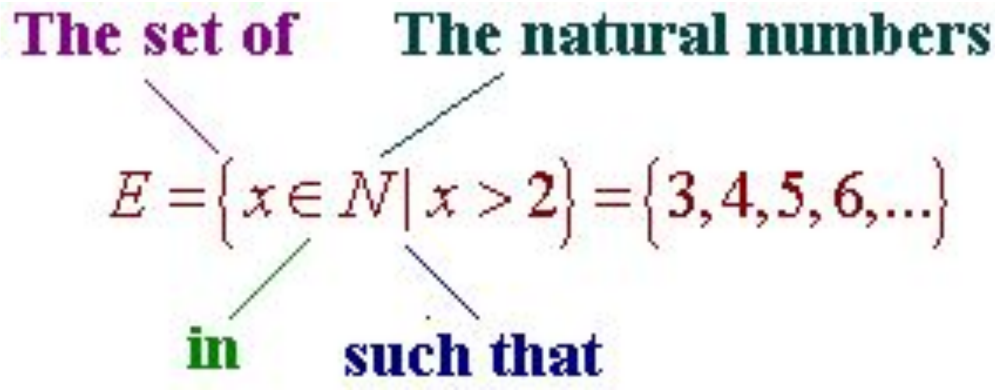
Property (or properties) of x that must be met in order to be an element of S

Set-Builder Notation



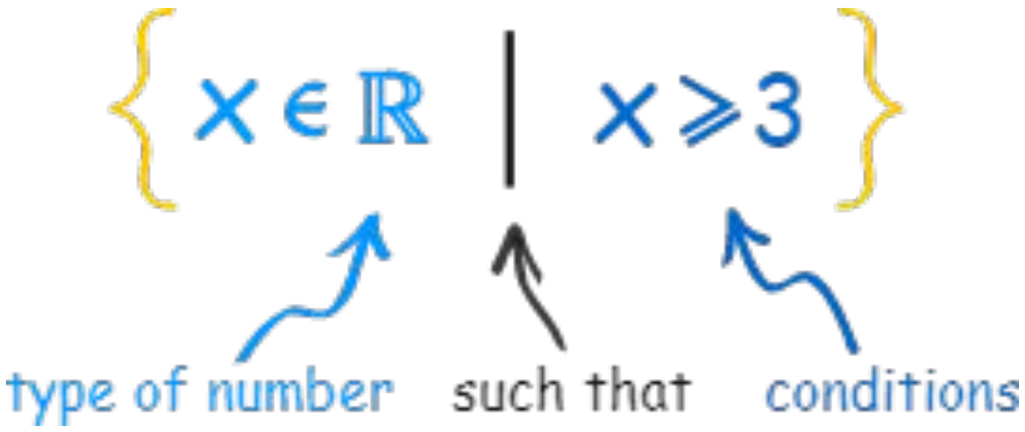
https://ltcconline.net/greenl/courses/152a/definitions/SETS.HTM

Set-Builder Notation



https://ltcconline.net/greenl/courses/152a/definitions/SETS.HTM

Set-Builder Notation



https://www.mathsisfun.com/sets/set-builder-notation.html

Set-Builder Notation

Let’s ***formalize*** our set operators in “set-builder notation”

Quick Side-Note:

-We will need to link together multiple “conditions” with “and’s”, “not’s” and “or’s”

Special symbols:

$\lor$ is “or” (notice similarity to $\cup$)

$\land$ is “and” (notice similarity to $\cap$)

$\neg$ is “not”

Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

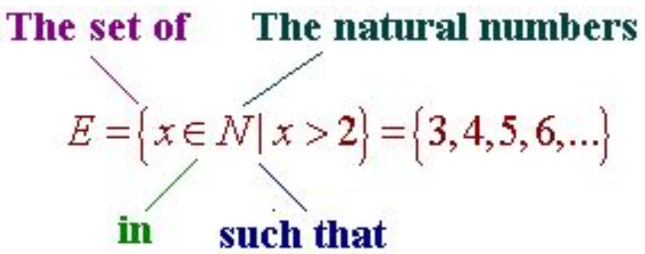
$$S \cap T =$$

For Reference:

$\vee$ is "or"

$\wedge$ is "and"

$\neg$ is "not"



Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

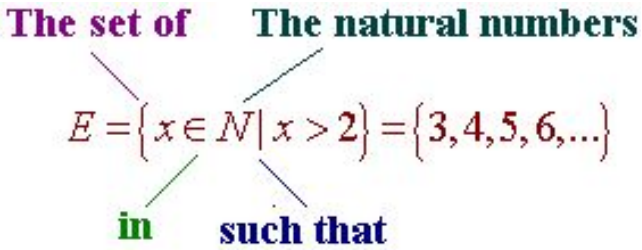
$$S \cap T = \{x \in U$$

For Reference:

∨ is "or"

∧ is "and"

¬ is "not"



Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

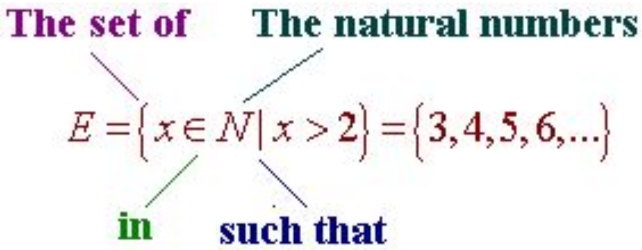
$$S \cap T = \{x \in U \mid x \in S$$

For Reference:

∨ is "or"

∧ is "and"

¬ is "not"



Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

$$S \cap T = \{x \in U \mid x \in S \wedge$$

For Reference:

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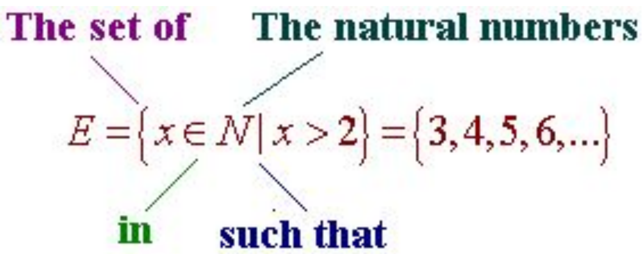

Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

$$S \cap T = \{x \in U \mid x \in S \wedge x \in T$$

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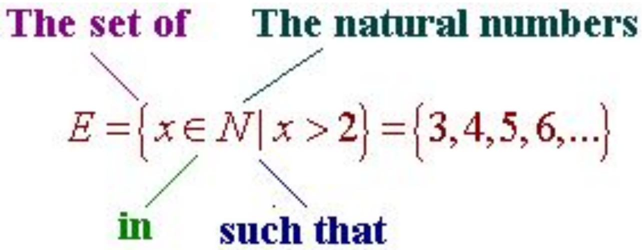

Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

$$S \cap T = \{x \in U \mid x \in S \wedge x \in T\}$$

For Reference:

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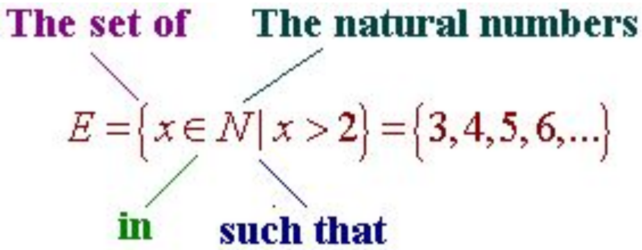
$\wedge$ is "and"

$\neg$ is "not"



Topics Expressed in Set-builder notation

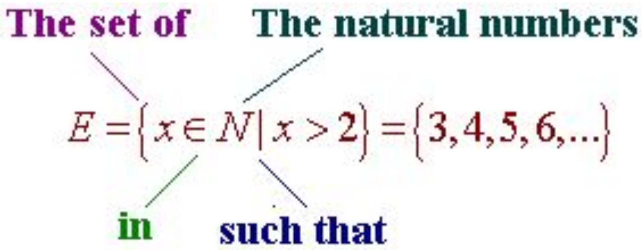
$$\boxed{\color{red}{?}} = \{x \in U \mid x \in S \land x \notin T\}$$

For Reference:

∨ is "or"

∧ is "and"

¬ is "not"



Topics Expressed in Set-builder notation

$$S \setminus T = \{x \in U \mid x \in S \wedge x \notin T\}$$

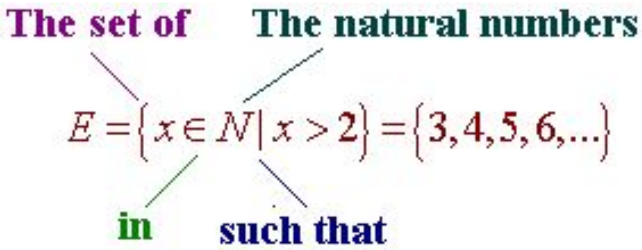
? $= \{x \in U \mid x \in S \vee x \in T\}$

For Reference:

∨ is "or"

∧ is "and"

¬ is "not"



Topics Expressed in Set-builder notation

$$S \setminus T = \{x \in U \mid x \in S \wedge x \notin T\}$$

$$S \cup T = \{x \in U \mid x \in S \vee x \in T\}$$

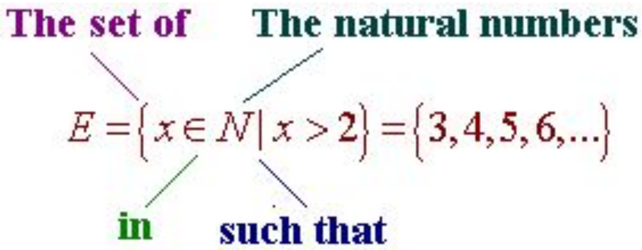
$$? = \{x \in U \mid x \in S \wedge x \in T\}$$

For Reference:

$\vee$ is "or"

$\wedge$ is "and"

$\neg$ is "not"



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? $= \{(a, b) \mid a \in A \text{ and } b \in B\}.$

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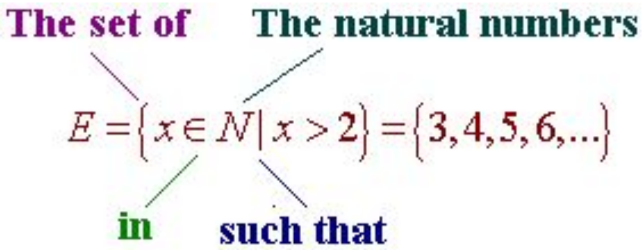

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$$S \cap T = \{x \in U \mid x \in S \wedge x \in T\}$$

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}.$$

$$? = \{ x \mid x \subseteq S \}$$

For Reference:

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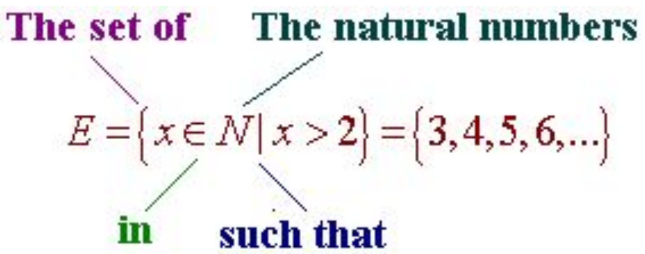

Topics Expressed in Set-builder notation

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$$S \cap T = \{x \in U \mid x \in S \wedge x \in T\}$$

A × B = {(a, b) | a ∈ A and b ∈ B}.

pow(S) = { x | x ⊆ S }

For Reference:

∨ is "or"

∧ is "and"

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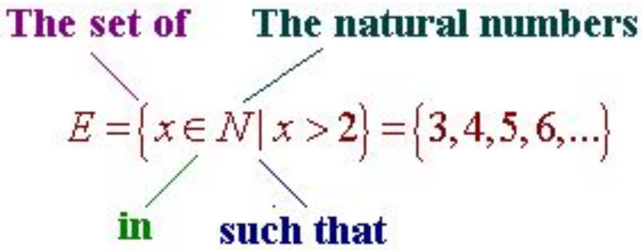

Set-builder Alert!

$X = \{1, 2, 3\}$, $Y = \{2, 3, 4\}$

Evaluate:

$$\{(a, b) \mid a \in X \text{ and } b \in Y\} = \underline{\hspace{6cm}}$$

For Reference:

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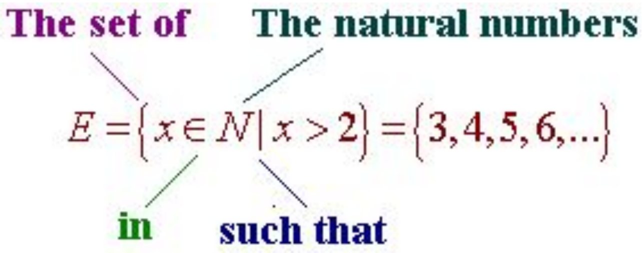


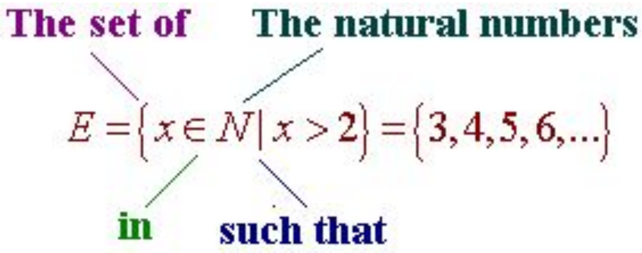
Set-builder Alert!

X = {1, 2, 3} , Y = {2, 3, 4}

Evaluate:

$\{\{a, b\} \mid a \in X \text{ and } b \in Y\} =$

$\{ \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2\}, \{2, 3\}, \{2, 4\}. \{3\}, \{3, 4\} \}$



For Reference:

$\vee$ is "or"