CS2120
Discrete Math
Jan 31st

Elizabeth Orrico & Chloe Smith

Agenda

- **Quiz Friday/Wknd**
- *Truth Tables*, sets, propositions

Ways to describe Sets

- Listing out the elements of a set works well for sets that are small and finite. What about larger sets?
- Set Builder Notation!

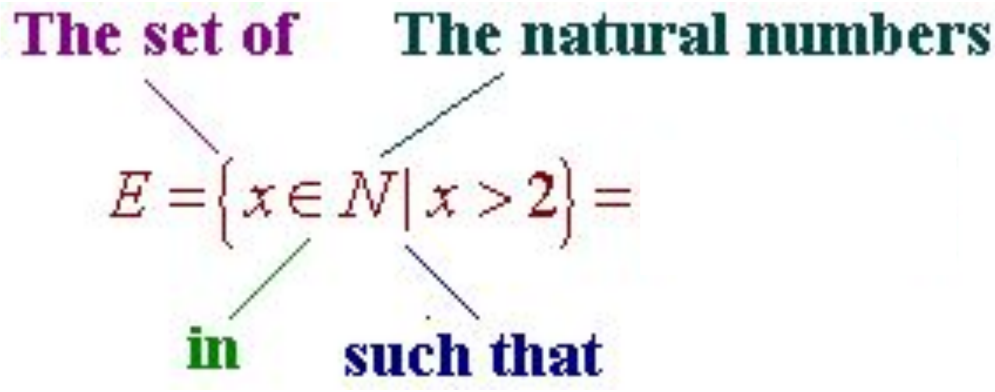
$$S = \{x \in A | x \text{ is blue}\}$$

The set of all x in A

Vertical Bar is read “such that”

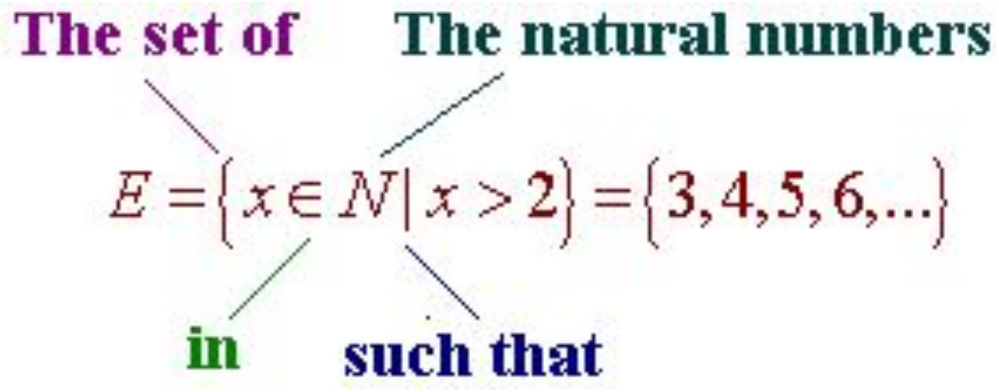
Property (or properties) of x that must be met in order to be an element of S

Set-Builder Notation



https://ltcconline.net/greenl/courses/152a/definitions/SETS.HTM

Set-Builder Notation



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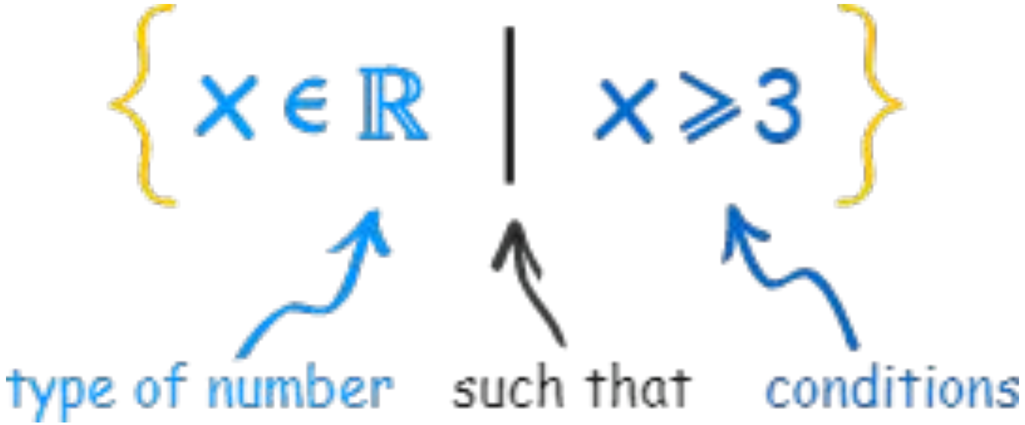


https://www.mathsisfun.com/sets/set-builder-notation.html

Set-Builder Notation

Let’s ***formalize*** our set operators in “set-builder notation”

Quick Side-Note:

-We will need to link together multiple “conditions” with “and’s”, “not’s” and “or’s”

Special symbols:

$\vee$ is “or” (notice similarity to $\cup$)

$\wedge$ is “and” (notice similarity to $\cap$)

$\neg$ is “not”

Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

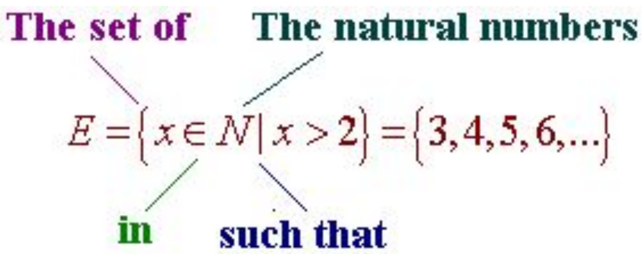
$$S \cap T =$$

For Reference:

$\vee$ is "or"

$\wedge$ is "and"

$\neg$ is "not"



Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

$$S \cap T = \{x \in U$$

For Reference:

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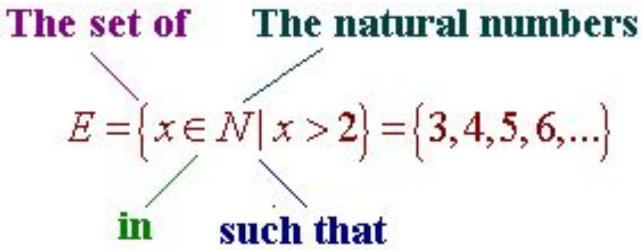

Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

$$S \cap T = \{x \in U \mid x \in S$$

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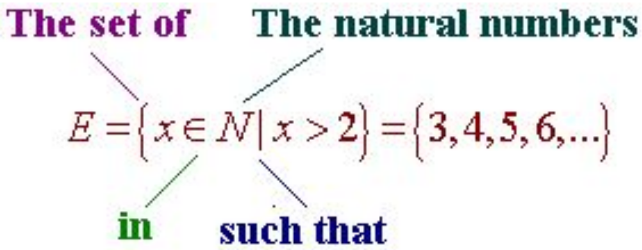

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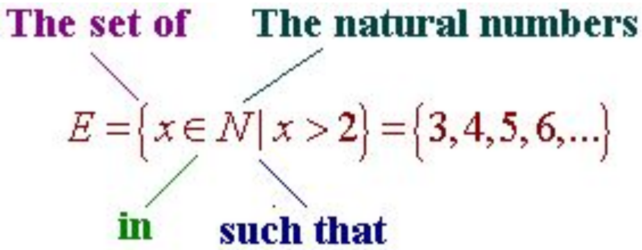

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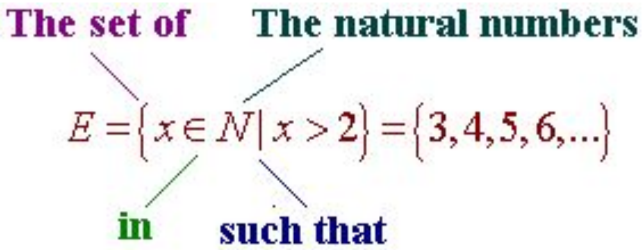

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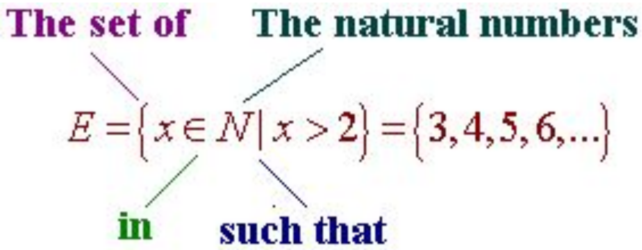

Topics Expressed in Set-builder notation

$$? = \{x \in U \mid x \in S \land x \notin T\}$$

For Reference:

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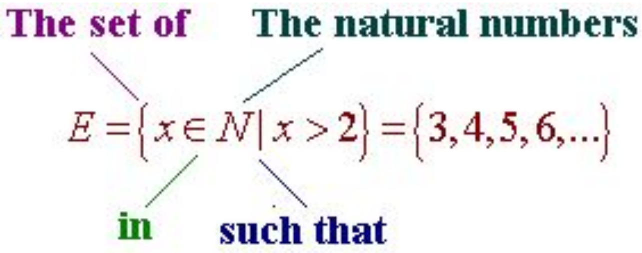

Topics Expressed in Set-builder notation

$$S \cup T = \{x \in U \mid x \in S \vee x \in T\}$$

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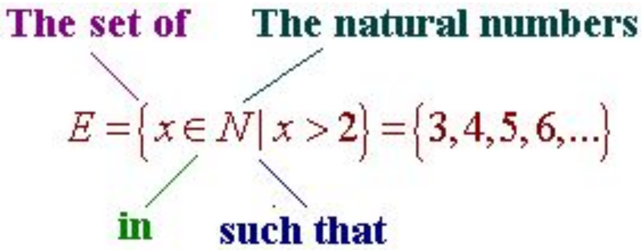

Topics Expressed in Set-builder notation

$$S \setminus T = \{x \in U \mid x \in S \wedge x \notin T\}$$

$$S \cup T = \{x \in U \mid x \in S \vee x \in T\}$$

$$? = \{x \in U \mid x \in S \wedge x \in T\}$$

For Reference:

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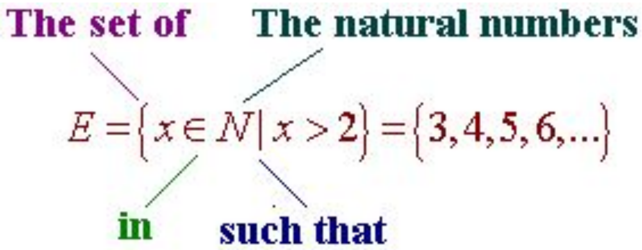

Topics Expressed in Set-builder notation

$$S \setminus T = \{x \in U \mid x \in S \land x \notin T\}$$

$$S \cup T = \{x \in U \mid x \in S \lor x \in T\}$$

$$S \cap T = \{x \in U \mid x \in S \land x \in T\}$$

? $= \{(a, b) \mid a \in A \text{ and } b \in B\}$.

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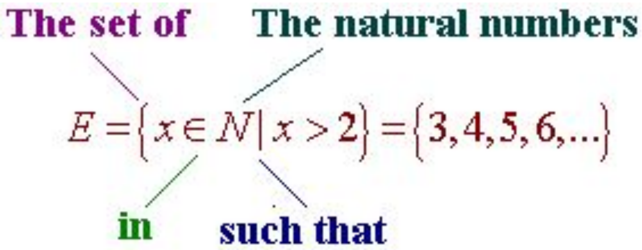

Topics Expressed in Set-builder notation

$$S \setminus T = \{x \in U \mid x \in S \wedge x \notin T\}$$

$$S \cup T = \{x \in U \mid x \in S \vee x \in T\}$$

$$S \cap T = \{x \in U \mid x \in S \wedge x \in T\}$$

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}.$$

$$? = \{ x \mid x \subseteq S \}$$

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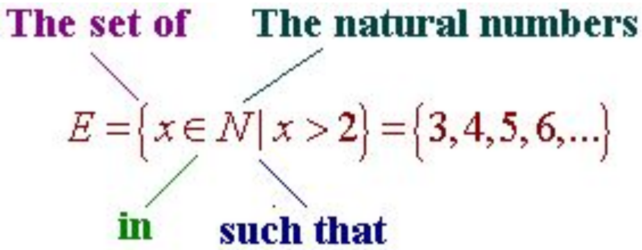

Topics Expressed in Set-builder notation

$$S \setminus T = \{x \in U \mid x \in S \wedge x \notin T\}$$

$$S \cup T = \{x \in U \mid x \in S \vee x \in T\}$$

$$S \cap T = \{x \in U \mid x \in S \wedge x \in T\}$$

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}.$$

$$\text{pow}(S) = \{ x \mid x \subseteq S \}$$

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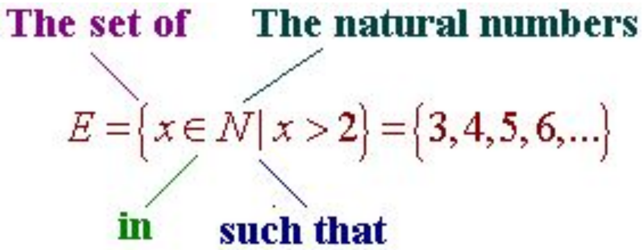

Set-builder Alert!

X = {1, 2, 3} ,   Y = {2, 3, 4}

Evaluate:

$$\{(a, b) \mid (a \in X) \land (b \in Y)\} = \underline{\hspace{6cm}}$$

For Reference:

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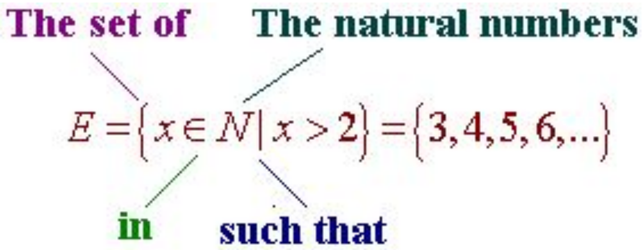


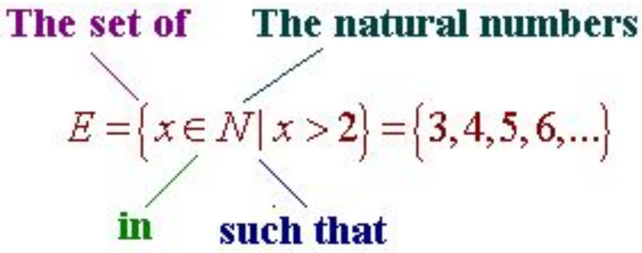
Set-builder Alert!

X = {1, 2, 3} ,   Y = {2, 3, 4}

Evaluate:

$$\{(a, b) \mid (a \in X) \land (b \in Y)\} = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\{ \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2\}, \{2, 3\}, \{2, 4\}, \{3\}, \{3, 4\} \}$$



For Reference:

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Propositions:

A proposition is a statement that is either true or false

Proposition Examples

(A proposition is a statement that is either true or false)

Examples of Propositions

"Turtles are plants"

"A proposition is a statement that is either true or false"

"The dress is black and blue"

"2 + 3 = 17"

Others?...

-
-
-

Proposition Examples

(A proposition is a statement that is either true or false)

Examples of Propositions

"Turtles are plants"

"A proposition is a statement that is either true or false"

"Jar Jar is a Sith Lord"

"The dress is black and blue"

"2 + 3 = 17"

(The others we listed on the last slide)

Examples of things that aren't Propositions

"What are you doing Friday?"

"Examples of things that aren't Propositions"

"What is 3 + 3?"

"Sit down!"

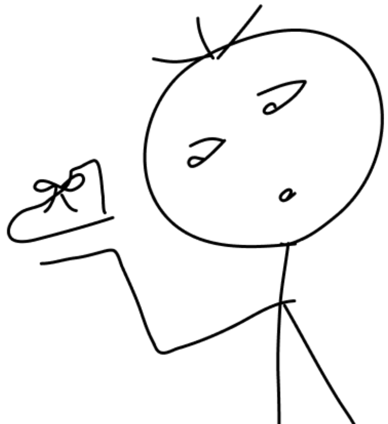
"yellow"

Propositions as Variables

(A proposition is a statement that is either true or false)

When dealing with propositions, we abstract away difficulties of defining, and we can just give them letters (define variables), like *p*

p = "Petty Pete stole my shoe"



Propositions as Truth Values

A proposition, *p,* is a statement that is either true or false. "True" or "False" is considered the "truth value" of *p*.

p = "Petty Pete stole my shoe" is either True or False

Propositions as Truth Values

A proposition, *p,* is a statement that is either true or false. "True" or "False" is considered the "truth value" of *p*.

Truth value can be written several ways:

Concept	Java/C	Python	This class	Bitwise	Other
true	`true`	`True`	$\top$ or 1	`-1`	T, tautology
false	`false`	`False`	$\bot$ or 0	`0`	F, contradiction

Propositions with Connectives

(A proposition is a statement that is either true or false)

We can modify, combine and relate propositions with *connectives*:

p = Pete stole my shoe, q = Pete bought ice cream

- $\lor$ is "or" — Pete stole my shoe OR Pete bought ice cream **$p \lor q$**
- $\land$ is "and" — Pete stole my shoe AND Pete bought ice cream **$p \land q$**
- $\neg$ is "not" — NOT THE CASE THAT Pete stole my shoe **$\neg p$**

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\vee$ is "or"
- $\wedge$ is "and"
- $\neg$ is "not"

$$\square = \{x \in U \mid x \in S \wedge x \notin T\}$$

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\vee$ is "or"
- $\wedge$ is "and"
- $\neg$ is "not"

$$S \setminus T = \{x \in U \mid x \in S \wedge x \notin T\}$$

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\vee$ is "or"
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Set theory is a branch of mathematical logic. So it makes sense to use logical language and symbols to describe sets.

Here you can reference what symbols we will use:

https://www.cs.virginia.edu/~emo7bf/cs2120/symbols.html

So far we have:

Concept	Java/C	Python	This class	Bitwise	Name	Other
true	`true`	`True`	$\top$ or 1	`-1`	tautology	T
false	`false`	`False`	$\bot$ or 0	`0`	contradiction	F
not P	`!p`	`not p`	$\neg P$ or $\overline{P}$	`~p`	negation	
P and Q	`p && q`	`p and q`	$P \wedge Q$	`p & q`	conjunction	PQ, $P \cdot Q$
P or Q	`p || q`	`p or q`	$P \vee Q$	`p | q`	disjunction	$P + Q$

“Not” operator

How to define:

Make a truth table

The anatomy of a truth table:

- **Column for each proposition**
- **Column for each connective within the entire statement**
- **Row for each combination of truth values**

Let's first look at some examples using the connectives "operators" that we have just discussed (or, and, not)!

“Not” operator

“Not” operator

p	$\neg p$
T	F
F	T

Note from anatomy:

-One proposition (p)

-One operator (¬)

-two possible truth value outcomes (p = T, p = F)

“And” operator

“And” operator

		And
P	Q	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

“Or” operator

“Or” operator

		Or
P	***Q***	$P \vee Q$
F	F	F
F	T	T
T	F	T
T	T	T

“Implies” operator

"Implies" operator

		Implies
P	***Q***	***P→Q***
F	F	T
F	T	T
T	F	F
T	T	T

“Xor” operator

“Xor” operator

		Xor
P	***Q***	$P \oplus Q$
F	F	F
F	T	T
T	F	T
T	T	F

“Bi-implication” operator

“Bi-implication” operator

		Bi-implies
P	***Q***	$P \leftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

In Summary…

		Or	*And*	*Implies*	*Xor*	*Bi-implies*
P	Q	$P \lor Q$	$P \land Q$	$P \rightarrow Q$	$P \oplus Q$	$P \leftrightarrow Q$
F	F	F	F	T	F	T
F	T	T	F	T	T	F
T	F	T	F	F	T	F
T	T	T	T	T	F	T

CS2120
Discrete Math
Feb 2nd

Elizabeth Orrico

Agenda

Quiz Friday - check out practice problems

- Power-sets
- Truth Tables

4.1.1 Some Popular Sets

Mathematicians have devised special symbols to represent some common sets.

symbol	**set**	**elements**
$\emptyset$	the empty set	none
$\mathbb{N}$	nonnegative integers	$\{0, 1, 2, 3, \ldots\}$
$\mathbb{Z}$	integers	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
$\mathbb{Q}$	rational numbers	$\frac{1}{2},\ -\frac{5}{3},\ 16,$ etc.
$\mathbb{R}$	real numbers	$\pi,\ e,\ -9,\ \sqrt{2},$ etc.
$\mathbb{C}$	complex numbers	$i,\ \frac{19}{2},\ \sqrt{2} - 2i,$ etc.

A superscript "$^+$" restricts a set to its positive elements; for example, $\mathbb{R}^+$ denotes the set of positive real numbers. Similarly, $\mathbb{Z}^-$ denotes the set of negative integers.

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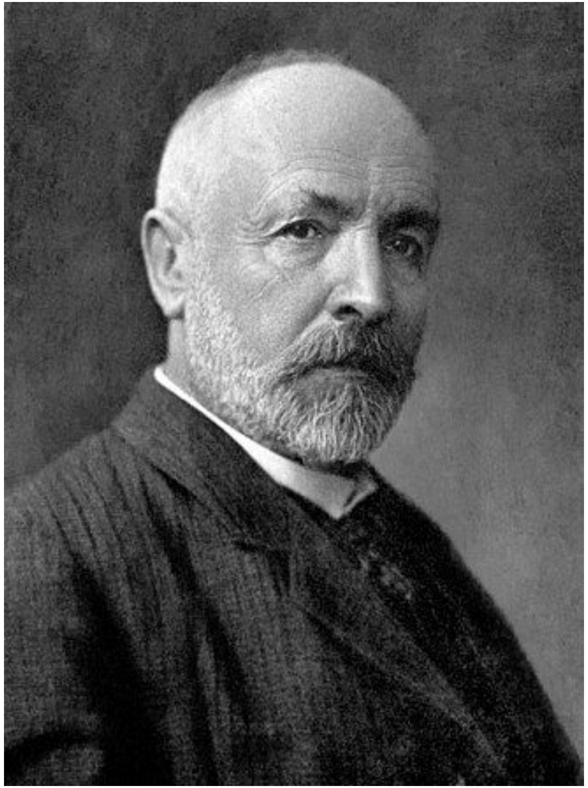
A superscript "+" restricts a set to its positive elements; for example, $\mathbb{R}^+$ denotes the set of positive real numbers. Similarly, $\mathbb{Z}^-$ denotes the set of negative integers.

Select $\mathbb{Z}^-$ from the choices given here

A: {0,1,2,3....} B: {1,2,3....} C: {....-3, -2, -1} D: {....-3, -2, -1, 0} E: None of the Above

Georg Ferdinand Ludwig Philipp Cantor ... was a German mathematician. **He created set theory,** which has become a fundamental theory in mathematics. Cantor] defined infinite and well-ordered sets, and proved that the real numbers are more numerous than the natural numbers. In fact, Cantor's method of proof of this theorem implies the existence of an infinity of infinities.



Cantor's work is of great philosophical interest, a fact he was well aware of. **Cantor's theory of transfinite numbers was originally regarded as so counter-intuitive – even shocking** – that it encountered resistance from mathematical contemporaries such as Leopold Kronecker and Henri Poincaré[3] and later from Hermann Weyl and L. E. J. Brouwer, while Ludwig Wittgenstein raised philosophical objections. Cantor, a devout Lutheran, **believed the theory had been communicated to him by God.**

The objections to Cantor's work were occasionally fierce: Leopold Kronecker's public opposition and personal attacks included describing Cantor as a **"scientific charlatan", a "renegade" and a "corrupter of youth".**

Power sets

4.1.3 Power Set

The set of all the subsets of a set, A, is called the *power set*, $\text{pow}(A)$, of A. So

$$B \in \text{pow}(A) \quad \text{IFF} \quad B \subseteq A.$$

For example, the elements of $\text{pow}(\{1,2\})$ are $\emptyset$, $\{1\}$, $\{2\}$ and $\{1,2\}$.

$$\mathcal{P}(S) = \{ x \mid x \subseteq S \}$$

Power sets

$$\mathcal{P}(S) = \{ x \mid x \subseteq S \}$$

where

Set A is a ***subset*** of set B

$$A \subseteq B$$

If & only if **all elements of A are also in B**

Power sets -- Your turn

For example, the elements of pow({1, 2}) are Ø, {1}, {2} and {1, 2}.

1.) What is the power-set of {}?

{ {} }

2.) What is the power set of {a, b, c}

{ {}, {a}, {b}, {c}, {a,b}, {a,c},{b,c},{a,b,c} }

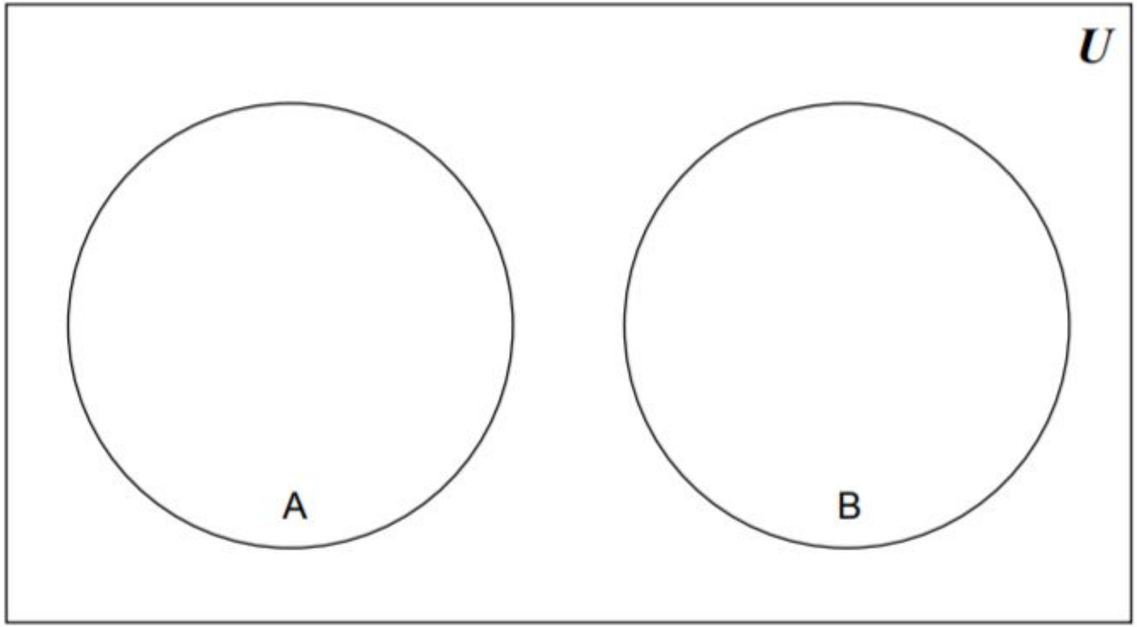
3.) What is the power set of { W, X, Y, Z }

Can we see a rule/pattern to determine the cardinality of a powerset?

Can we see a rule/pattern to determine the cardinality of a powerset?

$$|\mathcal{P}(X)| = 2^{|X|}$$

Disjoint Sets



Disjoint Sets

- Formal definition for disjoint sets: *two sets are disjoint if their intersection is the empty set*
- Further examples:
  - {1, 2, 3} and {3, 4, 5} are not disjoint

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  - {1, 2} and ∅ are disjoint
    - Their intersection is the empty set
  - ∅ and ∅ are disjoint!
    - Their intersection is the empty set

Set-builder Alert!

$X = \{1, 2, 3\}, \quad Y = \{2, 3, 4\}$

Evaluate:

$$\{\{a, b\} \mid (a \in X) \land (b \in Y)\} = \_\_\_\_\_\_\_\_\_\_\_\_$$

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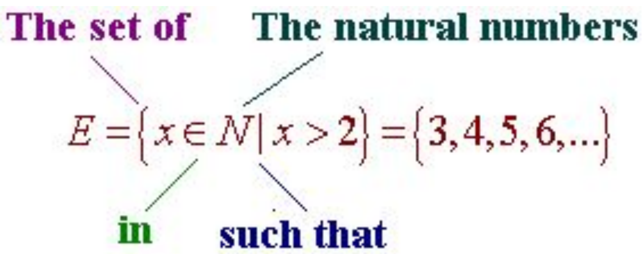

Set-builder Alert!

X = {1, 2, 3} ,   Y = {2, 3, 4}

Evaluate:

$$\{\{a, b\} \mid (a \in X) \wedge (b \in Y)\} = \_\_\_\_\_\_\_\_\_\_\_\_$$

$$\{\ \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2\}, \{2, 3\}, \{2, 4\}, \{3\}, \{3, 4\}\ \ \}$$

For Reference:

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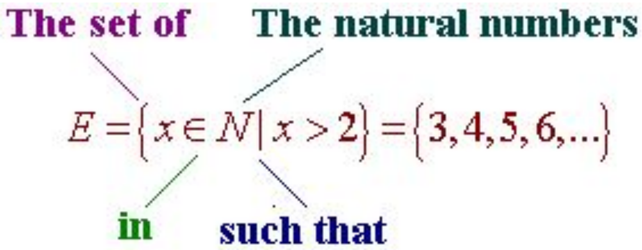

Propositions

A proposition, *p*, is a statement that is either true or false. "True" or "False" is considered the "truth value" of *p*.

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Concept	Java/C	Python	This class	Bitwise	Other
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Propositions

A proposition is a statement that is either true or false

We can combine and relate propositions with *connectives:*

		Or	***And***	***Implies***	***Xor***	***Bi-implies***
P	***Q***	$P \vee Q$	$P \wedge Q$	$P \rightarrow Q$	$P \oplus Q$	$P \leftrightarrow Q$
F	F	F	F	T	F	T
F	T	T	F	T	T	F
T	F	T	F	F	T	F
T	T	T	T	T	F	T

What if we want to combine logical operators for longer expressions?

Ex: $\neg (P \land Q)$

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

Apply the ∧ rule for the parentheses

P	Q	¬	(P	∧	Q)
T	T				
T	F				
F	T				
F	F				

First fill in the known values

P	Q	¬	(P	∧	Q)
T	T			T	
T	F				
F	T				
F	F				

Apply the ∧ rule for the parentheses

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬	(P	∧	Q)
T	T			T	
T	F			F	
F	T				
F	F				

Apply the ∧ rule for the parentheses

P	Q	$\neg(P \land Q)$
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	$\neg(P \land Q)$
T	T	T
T	F	F
F	T	F
F	F	

Apply the ∧ rule for the parentheses

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬(P ∧ Q)
T	T	T
T	F	F
F	T	F
F	F	F

Apply the ∧ rule for the parentheses

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬(P ∧ Q)
T	T	T
T	F	F
F	T	F
F	F	F

Apply the ∧ rule for the parentheses

P	Q	¬(P ∧ Q)
T	T	T
T	F	F
F	T	F
F	F	F

Apply the ¬ rule

P	Q	¬	(P	∧	Q)
T	T				
T	F				
F	T				
F	F				

First fill in the known values

P	Q	¬	(P	∧	Q)
T	T			T	
T	F			F	
F	T			F	
F	F			F	

Apply the ∧ rule for the parentheses

P	Q	¬	(P	∧	Q)
T	T	F		T	
T	F			F	
F	T			F	
F	F			F	

Apply the ¬ rule

P	Q	¬	(P	∧	Q)
T	T				
T	F				
F	T				
F	F				

First fill in the known values

P	Q	¬	(P	∧	Q)
T	T			T	
T	F			F	
F	T			F	
F	F			F	

Apply the ∧ rule for the parentheses

P	Q	¬	(P	∧	Q)
T	T	F		T	
T	F	T		F	
F	T			F	
F	F			F	

Apply the ¬ rule

P	Q	¬	(P	∧	Q)
T	T				
T	F				
F	T				
F	F				

First fill in the known values

P	Q	¬	(P	∧	Q)
T	T			T	
T	F			F	
F	T			F	
F	F			F	

Apply the ∧ rule for the parentheses

P	Q	¬	(P	∧	Q)
T	T	F		T	
T	F	T		F	
F	T	T		F	
F	F			F	

Apply the ¬ rule

P	Q	¬	(P ∧ Q)
T	T		
T	F		
F	T		
F	F		

First fill in the known values

P	Q	¬	(P ∧ Q)
T	T		T
T	F		F
F	T		F
F	F		F

Apply the ∧ rule for the parentheses

P	Q	¬	(P ∧ Q)
T	T	F	T
T	F	T	F
F	T	T	F
F	F	T	F

Apply the ¬ rule

P	Q	¬	(P	∧	Q)
T	T				
T	F				
F	T				
F	F				

First fill in the known values

P	Q	¬	(P	∧	Q)
T	T			T	
T	F			F	
F	T			F	
F	F			F	

Apply the ∧ rule for the parentheses

P	Q	¬	(P	∧	Q)
T	T	F		T	
T	F	T		F	
F	T	T		F	
F	F	T		F	

Apply the ¬ rule

Question 123

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Consider the expression "$(P \rightarrow Q) \leftrightarrow (Q \rightarrow P)$". This full expression has the same truth value as

A. ○ $P \oplus Q$

B. ○ $P \vee Q$

C. ○ $P \wedge Q$

D. ○ $P \rightarrow Q$

E. ○ $P \leftrightarrow Q$

F. ○ P

G. ◉ Q

▶ Key:

https://kytos.cs.virginia.edu/cs2102/quizzes/review.php?qid=1-1