CS2120
Discrete Math
Sept 13th

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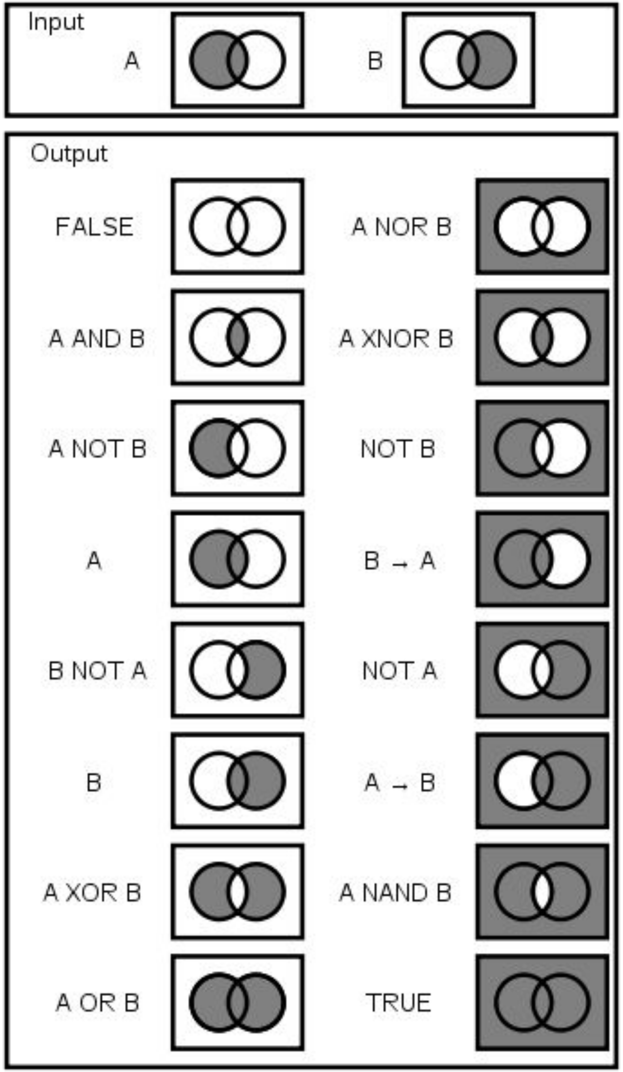
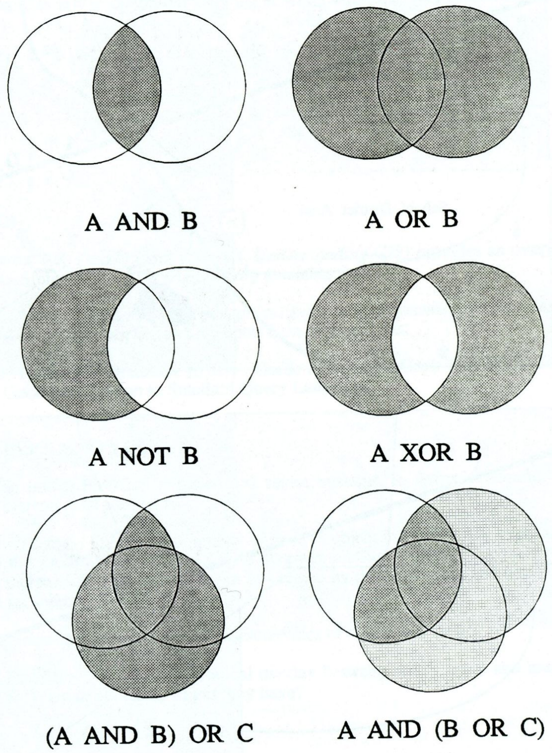
Agenda

- Quiz Friday
- Do Now
- A bit more on Implies
- DeMorgan's Law
- Everything you need: And's, Or's and Not's
- Product of Sums ⇔ Sum of Products
- What is 3SAT and why is it NP hard? Bonus! How does it relate to Set Cover?
- Equivalence Proofs
- What's Entailment?

Do NOW

- 18 years old & voted vs kid & voted


Input
A
B
Output
FALSE
A NOR B
A AND B
A XNOR B
A NOT B
NOT B
A
B → A
B NOT A
NOT A
B
A → B
A XOR B
A NAND B
A OR B
TRUE



Boolean Operators

Boolean Algebra

~~Boolean~~ Algebra

Prove $3(x + y) = 3x + 3y$

~~Boolean~~ Algebra

Prove	x	y	$3(x + y)$	$=$	$3x + 3y$

~~Boolean~~ Algebra

Prove	x	y	$3(x + y)$	=	$3x + 3y$
	0	0	$3(0 + 0)$	=	$3(0) + 3(0)$

~~Boolean~~ Algebra

Prove	x	y	$3(x + y)$	$=$	$3x + 3y$
	0	0	$3(0 + 0)$	$=$	$3(0) + 3(0)$
	1	0	$3(1 + 0)$	$=$	$3(1) + 3(0)$

~~Boolean~~ Algebra

Prove	x	y	$3(x + y)$	$=$	$3x + 3y$
	0	0	$3(0 + 0)$	$=$	$3(0) + 3(0)$
	1	0	$3(1 + 0)$	$=$	$3(1) + 3(0)$
	2	0	$3(2 + 0)$	$=$	$3(1) + 3(0)$

~~Boolean~~ Algebra

Prove	x	y	$3(x + y) = 3x + 3y$
	0	0	$3(0 + 0) = 3(0) + 3(0)$
	1	0	$3(1 + 0) = 3(1) + 3(0)$
	2	0	$3(2 + 0) = 3(2) + 3(0)$

No……...

“Is equivalent to”

$$\equiv$$

Double Negation

Prove $\neg\neg P \equiv P$

Double Negation

Prove $\neg\neg P \equiv P$

$\neg\neg P$	Given

Double Negation

Prove $\neg\neg P \equiv P$

$\neg\neg P$	Given
P	double negation

Double Negation

Prove $\neg\neg P \equiv P$

$\neg\neg P$	Given
$\equiv P$	double negation

Simplification

Prove $P \lor \bot \equiv P$

$P \lor \bot$	Given
$\equiv P$	Simplification

Simplification

Prove $P \equiv P \land (P \leftrightarrow T)$

Simplification

Prove $P \equiv P \land (P \leftrightarrow T)$

P	Given

Simplification

Prove $P \equiv P \land (P \leftrightarrow T)$

	P	Given
$\equiv$	$P \land P$	Simplification

Simplification

Prove $P \equiv P \land (P \leftrightarrow T)$

	P	Given
$\equiv$	$P \land P$	Simplification
$\equiv$	$P \land (P \leftrightarrow T)$	Simplification

Definition of Implication

Prove $A \rightarrow (B \oplus A) \equiv \neg A \vee (B \oplus A)$

Definition of Implication

Scratch work: $A \rightarrow B \equiv \neg A \vee B$ *where* $A = P$ *and* $B = (Q \oplus P)$

Prove $P \rightarrow (Q \oplus P) \equiv \neg P \vee (Q \oplus P)$

Definition of Implication

Scratch work: $A \rightarrow B \equiv \neg A \lor B$ where $A = P$ and $B = (Q \oplus P)$

Prove $P \rightarrow (Q \oplus P) \equiv \neg P \lor (Q \oplus P)$

$P \rightarrow (Q \oplus P)$	Given
$\equiv \neg P \lor (Q \oplus P)$	Definition of Implication

Definition of Implication

Prove $P \rightarrow (Q \oplus P) \equiv \neg P \lor (Q \oplus P)$

$P \rightarrow (Q \oplus P)$	Given
$\equiv \neg P \lor (Q \oplus P)$	Definition of Implication

Boolean Algebra

Associative Property: you can move parentheses around the operator

Example: $(2+3)+5 = 2+(3+5)$

Counterexample: $(2-3)-5 \neq 2-(3-5)$

Boolean Algebra

Which symbols are associative/commutative?

$\neg$ $\vee$ $\wedge$ $\oplus$ $\leftrightarrow$ $\rightarrow$

Associativity

Prove $(P \land Q) \land (A) \equiv P \land (Q \land (A))$

Associativity

Prove $(P \land Q) \land (R \lor Q) \equiv P \land (Q \land (R \lor Q))$

	$(P \land Q) \land (R \lor Q)$	Given
$\equiv$	$P \land (Q \land (R \lor Q))$	Associativity

Boolean Algebra

Commutative Property: you can swap their operands' position

Example: $2+3 = 3+2$

Counterexample: $2-3 \neq 3-2$

Boolean Algebra

Which symbols are associative/commutative?

$\neg \quad \vee \quad \wedge \quad \oplus \quad \leftrightarrow \quad \rightarrow$

Commutativity

Prove $(P \lor Q) \lor (R \lor Q) \equiv (P \lor Q) \lor R$

Commutativity

Prove $(P \vee Q) \vee (R \vee Q) \equiv (P \vee Q) \vee R$

	$(P \vee Q) \vee (R \vee Q)$	Given
$\equiv$	$(P \vee Q) \vee (Q \vee R)$	Commutativity
$\equiv$	$(P \vee (Q \vee Q)) \vee R$	Associativity
$\equiv$	$(P \vee Q) \vee R$	Simplification

DeMorgan’s Law

Prove $P \lor \neg(Q \land \neg R) \equiv P \lor (\neg Q \lor R)$

DeMorgan's Law

Prove $P \lor \neg(Q \land \neg R) \equiv P \lor (\neg Q \lor R)$

	$P \lor \neg(Q \land \neg R)$	Given
$\equiv$	$P \lor (\neg Q \lor \neg\neg R)$	DeMorgan's Law
$\equiv$	$P \lor (\neg Q \lor R)$	Double Negation

Distributive Law

Prove $(P \lor \neg Q) \land (P \lor \neg R) \equiv P \lor (\neg Q \land \neg R)$

Distributive Law

$A \lor (B \land C) = (A \lor B) \land (A \lor C)$ $A = P$ $B = \neg Q$ $C = \neg R$

Prove $(P \lor \neg Q) \land (P \lor \neg R) \equiv P \lor (\neg Q \land \neg R)$

Distributive Law

$A \lor (B \land C) = (A \lor B) \land (A \lor C)$ $A = P$ $B = \neg Q$ $C = \neg R$

Prove $(P \lor \neg Q) \land (P \lor \neg R) \equiv P \lor (\neg Q \land \neg R)$

	$(P \lor \neg Q) \land (P \lor \neg R)$	Given
$\equiv$	$P \lor (\neg Q \land \neg R)$	Distributive Law