February 16 Slides

So far…

1. We’ve given you an introduction to **classical logic**, which consists of…
    a. Propositional Logic
    b. Predicate Logic (a.k.a First Order Logic)
2. **Propositional Logic** deals with
    a. **Propositions**: Sentences that evaluate to either true or false.
    b. **Logical Operators**: **→, ↔, ¬, ∧, ∨, ⊕**
3. We’ve also started introducing you to **Predicate Logic**
    a. **Predicates**: “A function that evaluates to True or False”
        i. E.g. R(x) = x is red.
        ii. You can also think of it in grammatical terms.
            1. English sentences have subjects and predicates.
    b. Quantifiers: ∀ and ∃

Convert this deductive argument into Propositional Logic:

1) 6 is an even number.
2) All real numbers that are even are natural numbers.
C1) 6 is a natural number.

As we can see, it's not very useful to convert this to propositional logic.

P: 6 is an even number.
Q: All real numbers that are even are natural numbers.
R: 6 is a natural number.

1) P
2) Q
C1) R

Now let’s do this with Predicate Logic:

s: 6
E(x) = x is an even number
N(x) = x is a natural number
Domain: Real Numbers

1) E(s)
2) $\forall x.\ E(x) \rightarrow N(x)$
C1) N(s)

Universal Quantifiers and Conjunctions.

s: 6
E(x) = x is an even number
N(x) = x is a natural number
Domain: Real Numbers

$\forall x.\ E(x) \rightarrow N(x)$ *All real numbers that are even are natural numbers.*

$\forall x.\ E(x) \land N(x)$ *All real numbers are even numbers and natural numbers.*

NOT EQUIVALENT! Though it's possible, you typically won't see sentences like the second statement.

Existential Quantifiers and Implication.

s: 6
E(x) = x is an even number
N(x) = x is a natural number
Domain: Real Numbers

$\exists x.\ E(x) \land N(x)$ *There exists a real number that is even and a natural number.*

$\exists x.\ E(x) \rightarrow N(x)$ *There exists a real number that makes the statement true: if the number is an even number, then it is a natural number.*

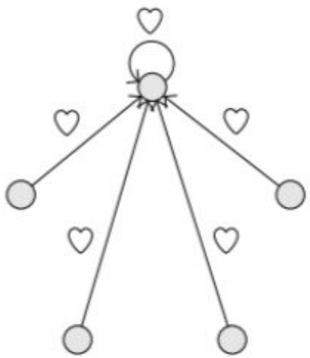
NOT EQUIVALENT! Though it's possible, you typically won't see sentences like the second statement because it's vacuously true, meaning the statement would still hold true even if E(x) is false. (The statement would still be true if the domain were to be the odd numbers.)

Quantifiers Review

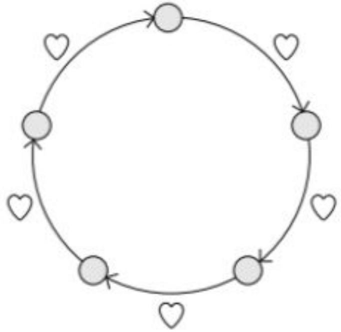
Domain: People   L(x, y) = *x loves y*

Are these equivalent?

$\exists y. \forall x.\ L(x,y)$



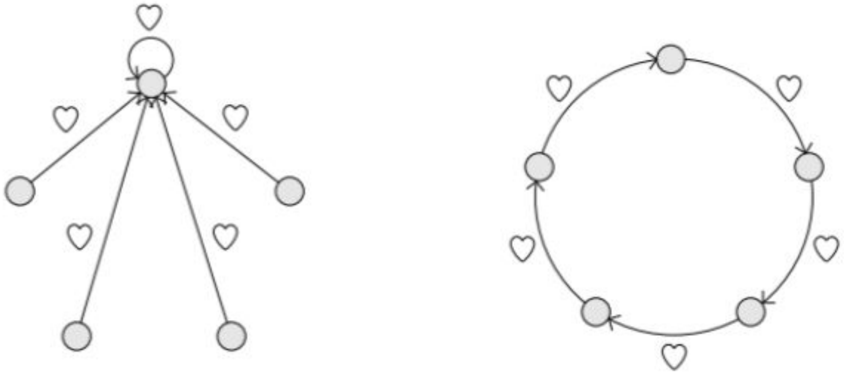
$\forall x. \exists y.\ L(x,y)$



Quick Intro to Multiple Quantifiers:

Domain: People   L(x, y) = *x loves y*
Are these equivalent?

$\exists y \forall x\ L(x,y)$ **is not equivalent to** $\forall x \exists y\ L(x,y)$



A Note on Notation.

$$\forall z.(\exists y.A(y) \vee E(y)) \to A(z)$$

- What is the period?
- The period is a replacement for parentheses which tell us the scope of the quantifiers. Periods tell us that the scope of the quantifier is starting from it to the end of the statement or a closing parenthesis to an open parenthesis that came before the quantifier (whichever comes first left to right).

$$\forall z[(\exists y[A(y) \vee E(y)]) \to A(z)]$$

∃ and ∀

Associate "for all" with AND's since it becomes false if just one truth value is false.

Associate "there exists" with OR's since it becomes true if just one truth value is true.

Think about boolean logic

Domain: {Ann, Bob, Chris} $\exists y \forall x \, L(x,y)$

$$
\begin{aligned}
&\quad (\ L(\text{Ann}, \text{Ann}) \land L(\text{Bob}, \text{Ann}) \land L(\text{Chris}, \text{Ann})\ ) \\
\lor &\quad (\ L(\text{Ann}, \text{Bob}) \land L(\text{Bob}, \text{Bob}) \land L(\text{Chris}, \text{Bob})\ ) \\
\lor &\quad (\ L(\text{Ann}, \text{Chris}) \land L(\text{Bob}, \text{Chris}) \land L(\text{Chris}, \text{Chris})\ )
\end{aligned}
$$

Think about boolean logic

Domain: {Ann, Bob, Chris} $\exists y \forall x\ L(x,y)$

How will this change for "$\forall x \exists y\ L(x,y)$"?

$$
\begin{aligned}
& (\ L(Ann, Ann) \land L(Bob, Ann) \land L(Chris, Ann)\ ) \\
\lor\ & (\ L(Ann, Bob) \land L(Bob, Bob) \land L(Chris, Bob)\ ) \\
\lor\ & (\ L(Ann, Chris) \land L(Bob, Chris) \land L(Chris, Chris)\ )
\end{aligned}
$$

Think about boolean logic

Domain: {Ann, Bob, Chris} $\exists y.\ \forall x.\ L(x,y) = \exists y(\forall x\ (L(x,y)))$

How will this change for "$\forall x\ \exists y\ L(x,y)$"?

$$
\begin{aligned}
& (\ L(Ann, Ann) \lor L(Ann, Bob) \lor L(Ann, Chris)\ ) \\
\land\ & (\ L(Bob, Ann) \lor L(Bob, Bob) \lor L(Bob, Chris)\ ) \\
\land\ & (\ L(Chris, Ann) \lor L(Chris, Bob) \lor L(Chris, Chris)\ )
\end{aligned}
$$

Think about nested loops

Domain: {Ann, Bob, Chris} $\exists y \forall x\, L(x,y)$

```
// since ∃ means stuff “or’d” together, start with false
existValue = False
for y in {Ann, Bob, Chris}:
    // since ∀ means stuff “and’d” together, start with
true
    univValue = True
    for x in {Ann, Bob, Chris}:
        univValue = univValue ∧ L(x,y)
    end
    existValue = existValue ∨ univValue
end
Return existValue
```

Think about nested loops

Domain: {Ann, Bob, Chris} $\exists y \forall x\, L(x,y)$

How will this code change for "$\forall x \exists y\, L(x,y)$"?

```
// since ∃ means stuff "or'd" together, start with false
existValue = False
for y in {Ann, Bob, Chris}:
    // since ∀ means stuff "and'd" together, start with
true
    univValue = True
    for x in {Ann, Bob, Chris}:
        univValue = univValue ∧ L(x,y)
    end
    existValue = existValue ∨ univValue
end
Return existValue
```

Think about nested loops

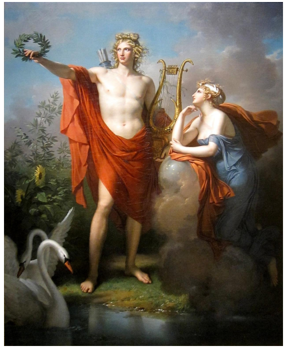
Domain: {Ann, Bob, Chris} $\forall x \exists y\, L(x,y)$

```
// since ∀ means stuff “and’d” together, start with true
univValue = True
for x in {Ann, Bob, Chris}:
    // since ∃ means stuff “or’d” together, start with false
    existValue = False
    for y in {Ann, Bob, Chris}:
        existValue = existValue ∨ L(x,y)
    end
    univValue = existValue ∧ univValue
end
Return univValue
```

Domain: People

$F = \{Apollo, Britomartis, Cupid, Demeter, Bob\}$

$$\mathbf{L(x, y) =} \ \boldsymbol{x \text{ loves } y}$$



$$\forall x \in F.\ \forall y \in F. L(x,y)$$

Negating Quantifiers

- If in this form ($\nexists$), convert to ($\neg\exists$)
- Swap the quantifier ($\neg\forall$ becomes $\exists$) or ($\neg\exists$ becomes $\forall$)
- Distribute the negation to whatever is inside the scope of the quantifier.
  - $\neg\exists x.(F(x) \vee A(x))$ becomes $\forall x.\neg(F(x) \vee A(x))$

Domain: People

F = {Apollo, Britomartis, Cupid, Demeter, Bob}

L(x, y) = *x loves y*

$\forall x \in F\ (\ \forall y \in F\ (L(x,y)))\quad \equiv \quad \forall x \in F.\ \forall y \in F.\ L(x,y)$

DeMorgan's Practice: $\neg(\neg\forall\ x,y \in F.\ L(x,y))$

Domain: People

F = {Apollo, Britomartis, Cupid, Demeter, Bob}

L(x, y) = *x loves y*

$\forall x \in F\ (\ \forall y \in F\ (L(x,y))) \quad \equiv \quad \forall x \in F.\ \forall y \in F.\ L(x,y) \quad \equiv \quad \forall\ \boldsymbol{x,y \in F.\ L(x,y)}$

$\neg(\neg\forall\ \mathbf{x,y \in F.\ L(x,y)})$

Domain: People

$F = \{Apollo, Britomartis, Cupid, Demeter, Bob\}$

L(x, y) = *x loves y*

$\forall x \in F\ (\ \forall y \in F\ (L(x,y)))\quad \equiv \quad \forall x \in F.\ \ \forall y \in F.\ L(x,y) \quad \equiv \quad \boldsymbol{\forall\ x,y \in F.\ L(x,y)}$

DeMorgan's Practice: $\neg(\neg\forall\ x,y \in F.\ L(x,y)) \quad \equiv \quad$ ***<u>????</u>***

Domain: People

F = {Apollo, Britomartis, Cupid, Demeter, Bob}

L(x, y) = *x loves y*

$\forall x \in F\ (\ \forall y \in F\ (L(x,y)))\quad \equiv \quad \forall x \in F.\ \forall y \in F.\ L(x,y) \quad \equiv \quad \boldsymbol{\forall\ x,y \in F.\ L(x,y)}$

DeMorgan's Practice: $\neg(\neg \forall\ x,y \in F.\ L(x,y)) \equiv \neg(\ \exists x \in F.\ \neg(\ \forall y \in F.\ L(x,y)\ ))$

Domain: People

$F = \{Apollo, Britomartis, Cupid, Demeter, Bob\}$

$$\mathbf{L(x, y)} = \boldsymbol{x\ loves\ y}$$

$$\forall x \in F\ (\ \forall y \in F\ (L(x,y))) \quad \equiv \quad \forall x \in F.\ \ \forall y \in F.\ L(x,y) \quad \equiv \quad \boldsymbol{\forall\ x,y \in F.\ L(x,y)}$$

DeMorgan's Practice: $\boldsymbol{\neg(\neg\ \forall\ x,y \in F.\ L(x,y))} \equiv \neg(\ \exists x \in F.\ \neg(\ \forall y \in F.\ L(x,y)\ ))$

$$\equiv \neg(\ \exists x \in F.\ \exists y \in F.\ \neg(\ L(x,y)\ ))$$

Domain: People

$F = \{Apollo, Britomartis, Cupid, Demeter, Bob\}$

L(x, y) = *x loves y*

$\forall x \in F\ (\ \forall y \in F\ (L(x,y)))\quad \equiv \quad \forall x \in F.\ \ \forall y \in F.\ L(x,y) \quad \equiv \quad \boldsymbol{\forall\ x,y \in F.\ L(x,y)}$

DeMorgan's Practice: $\boldsymbol{\neg(\neg \forall\ x,y \in F.\ L(x,y))}\ \equiv \neg(\ \exists x \in F.\ \neg(\ \forall y \in F.\ L(x,y)\ ))$

$\equiv \neg(\ \exists x \in F.\ \ \exists y \in F.\ \neg(\ L(x,y)\ ))$

$\equiv \neg(\ \exists x \in F.\ \ \exists y \in F.\ \neg L(x,y)\ )$

Domain: People

$F = \{Apollo, Britomartis, Cupid, Demeter, Bob\}$

L(x, y) = *x loves y*

$\forall x \in F\ (\ \forall y \in F\ (L(x,y)))\quad \equiv \quad \forall x \in F.\ \ \forall y \in F.\ L(x,y) \quad \equiv \quad \boldsymbol{\forall\ x,y \in F.\ L(x,y)}$

DeMorgan's Practice: $\boldsymbol{\neg(\neg \forall\ x,y \in F.\ L(x,y))} \equiv \neg(\ \exists x \in F.\ \neg(\ \forall y \in F.\ L(x,y)\ ))$

$\equiv \neg(\ \exists x \in F.\ \ \exists y \in F.\ \neg(\ L(x,y)\ ))$

$\equiv \neg(\ \exists x \in F.\ \ \exists y \in F.\ \neg(\ L(x,y)\ ))$

$\equiv \neg(\ \exists x,y \in F.\ \neg L(x,y)\ )$