Name: ______

CompID: _____

Theorem 1
$$\left(1 + \sum_{i=0}^{n-1} 2^i\right) = 2^n$$

and $E = \{(v, v+1) \mid (v \in V) \land (v+1 \in V)\}$ has $\left(\sum_{i=1}^{n-1} i\right)$ distinct paths

PROBLEM 1 Prove one of theorems 1–2 (your choice which one) using induction

Name:		_ CompID:		
Theorem 1 $\log_5(7) \notin \mathbb{Q}$	Theorem 2 $\sqrt[3]{11} \notin \mathbb{Q}$	Theorem 3 $\sum_{i=0}^{n} 2^{i} = 2^{n+1} - 1$		
PROBLEM 1 Prove one of theorems 1–3 (your choice which one) using contradiction				

Name: _____

CompID: _____

PROBLEM 1 Fill in these **combinatorics** blanks

You may answer any question with factorial, choose, and unresolved arithmetic notation, but may not use

ellipses. For example, the following are all OK: 120 , $5!$,	$\frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{(2\cdot 1)+(3\cdot 2\cdot 1)},$	$\begin{pmatrix} 5\\ 3 \end{pmatrix}$	
---	---	---------------------------------------	--

An economy license plate number starts with X or W, then two more letters (out of 24 options, not 26, because O and I are not used), then four digits (all ten used). Repetition is allowed (e.g., "WWX 0000" is OK). How many license plate numbers can this scheme create?

2. _____ How many 9-element subsets of a 100-element set are there?

3. _____ I draw 3 cards from a deck of 50 distinct cards and line them up in a row. How many distinct rows of cards could I get?

4. _____ Which is larger:
$$\begin{pmatrix} 45\\40 \end{pmatrix}$$
 or $\begin{pmatrix} 45\\42 \end{pmatrix}$?

^{5.} _____ A palindrome is a string is the same if you reverse it, like "rrynyrr". How many 7-letter palindromes can be made from the set of 26 letters?

6. ______ I randomly shuffle a list containing five "d"s, five "q"s, and five "w"s,. What is the probability the shuffle will result in the exact sequence "dqwdqwdqwdqwdqwd?

^{7.} I randomly shuffle a 15-item list containing the integers 1 through 15. What is the probability the shuffle will result in the exact sequence (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)?

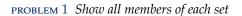
^{8.}_____ I roll a pair of twenty-sided dice, with sides numbered 1 through 20. What is the probability at least one die will roll a 6?

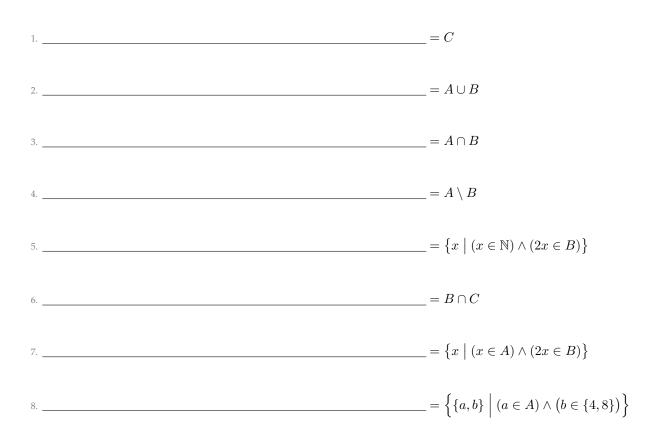
9. ______You can either choose a 5-letter string (out of 26 letters, which can repeat; e.g "xyxxy" or dfghj are both options) or a 5-digit number (out of ten digits, which can repeat, but the first digit must not be 0; e.g. 21020 is an option but 02102 is not). How many options do you have?

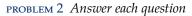
Name: _____

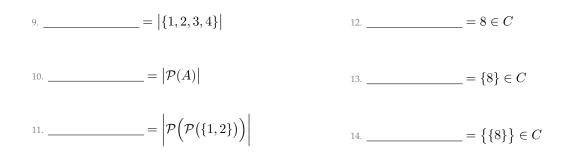
CompID: _____

Consider the following sets: $A = \{2, 4, 8\}, B = \{1, 2, 4\}, C = \mathcal{P}(\{1, 2\})$









Name:	CompID:	
Consider the following discrete structure	s questions.	
PROBLEM 1 Write out in full		
1	$_ = \{1, 2\} \times \{1\} \times \{1, 2\}$	
2	$_{-} = \{1, 2, 3\}^4 $	
3	$_{-}$ = all the subsequences of "OK"	
4	_ = a subsequence of "xyxxy" that is not a substring of "xyxxy"	
5	_ = the image of $\{0,3\}$ under $R(x) = x + 2$	
6	_ = the set of edges of the graph $① \rightarrow 2 \rightleftharpoons 3$	
PROBLEM 2 Draw		
7. $\bigcirc \xrightarrow{\leftarrow} \bigcirc \leftarrow \bigcirc$ add a minimal number	of edges to make this the graph of a transitive relation	
8. $\bigcirc \xrightarrow{\leftarrow} \bigcirc \leftarrow \bigcirc$ add a minimal number	of edges to make this the graph of a symmetric relation	
PROBLEM 3 Logarithms		
9. Simplify $\log_5(24) - \log_5(4)$:		
10. Re-write $\log_{10}(x^7)$ without exponentiation:		
11. Re-write $\log_w(8)$ using base- $x \log(s)$ instead of base- w :		
12. Fill in the blank: $\log_9(4) = \log_3\left($)	

Name: _____

CompID: _____

Consider the following **logic** questions. You do not need to specify your domains, propositions, or predicate definitions, though you may if you wish.

PROBLEM 1 Convert the underlined parts to logic

1. "can any sorting algorithm be faster than quicksort?"

2. "Every klaxon is a horn."

3. "No patents apply to this code"

4. "Some 100-digit number is prime."

PROBLEM 2 Convert to English

 $\begin{array}{l} \textbf{domain: all animals} \\ M(x): ____x \text{ is a monkey} \\ L(x,y): ____x \text{ loves }___y \\ \text{5. Write a clear English sentence that means } \exists x \ . \ \forall y \neq x \ . \ L(y,x). \end{array}$

6. Write a clear English sentence that means $\forall x : M(x) \rightarrow (\exists y : L(x,y) \land L(y,x))$.

(continued on reverse)

PROBLEM 3 Apply axioms

Show that $((P \land Q) \rightarrow W) \equiv (P \rightarrow (W \lor \neg Q))$ by direct proof and/or proof by cases. You may mix math and English if you wish; we are looking for sound logic, not prose proof technique.