Name: ______________________________ CompID: ______________

CS 2102 - DMT1 - Fall 2019 — Luther Tychonievich
Administered in class friday october 18, 2019

Quiz 06

This quiz refers to the following two definitions. The first we've seen before:

Definition 1 (Natural Number) *The number 0 is a natural number. The number 1 is a natural number. For any two natural numbers, x and y, their sum $x + y$ is also a natural number. There are no other natural numbers.*

The second is often left undefined, relying on an intuitive sense of what it means to be finite. However, having a few rules might help:

Definition 2 (Finite) *Any integer you can write (and finish writing) in base-10 is finite. The sum or product of two finite numbers is finite. Any number whose absolute value is smaller than some finite number is finite.*

problem 1 *Proof by Induction*

Complete the following proof by induction of the theorem "every natural number is finite."

Proof. We use induction. The induction hypothesis is that a number is finite.

Base case: 0 ______________ is finite, because

it can be written using digits (i.e., "0").

Inductive step: Assume that some $n \in \mathbb{N}$ ______________ is finite. Then $n + 1$ ______________ must be finite because

$n + 1$ is the sum of two finite numbers: n is finite by the inductive hypothesis, and 1 is finite because it can be written using digits (i.e., "1").

It follows by induction that all natural numbers are finite. □

PROBLEM 2 *Proof by Contradiction*

Complete the following proof by contradiction of the theorem "there is no largest natural number."

Proof. Assume there was a largest natural number, $m \in \mathbb{N}$, where $\nexists n \in \mathbb{N} \,.\, n > m$.
Consider the number $x > m$ defined as

$m + 1$

We know that $x \in \mathbb{N}$ because

x is the sum of two natural numbers (m and 1).

But that contradicts $\nexists n \in \mathbb{N} \,.\, n > m$.
Because

assuming there was a largest natural number led to a contradiction,

there is no largest natural number. □